Modelling the effective elastic moduli of partially saturated porous rocks

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CFMR
COMITÉ FRANÇAIS DE MÉCANIQUE DES ROCHES

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sktodowska-Curie grant agreement No 101007851

Outline:

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- Motivation

- Mesoscopic Wave-Induced Fluid Flow:
  - ✓ Physical process
  - ✓ Poroelastic modelling
  - ✓ Case study: *Hysteresis effects*

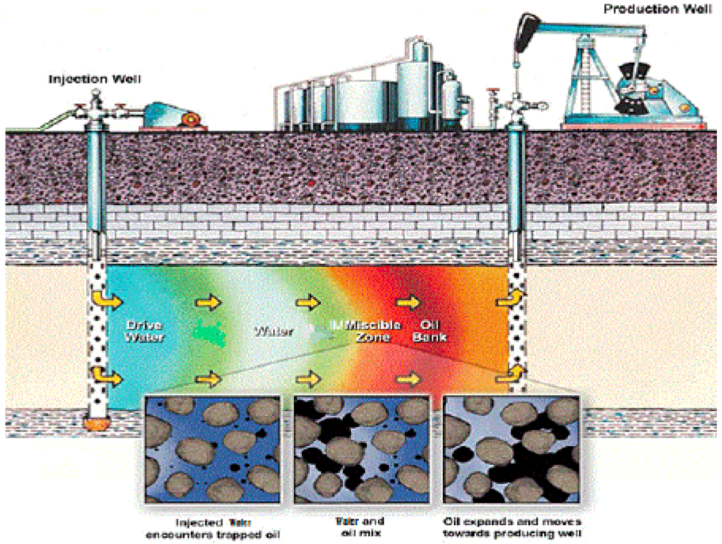
- Squirt flow:
  - ✓ Physical process
  - ✓ Modelling approaches
  - ✓ Case study: *Partially saturated crack*

- Biot's intrinsic mechanism:
  - ✓ Physical process
  - ✓ Biot's frequency
  - ✓ Case study: *Dynamic permeability*

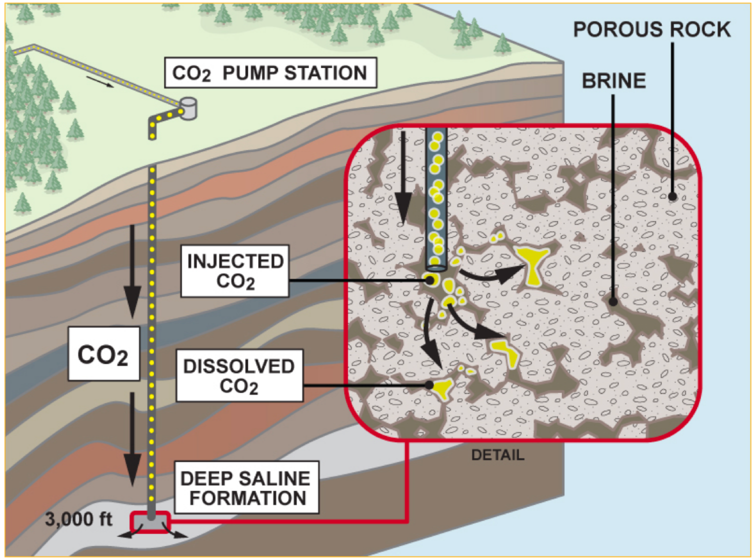
- Final remarks

Motivation

Enhanced oil recovery

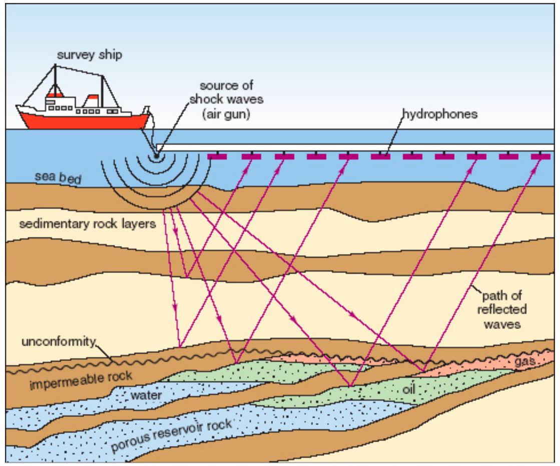


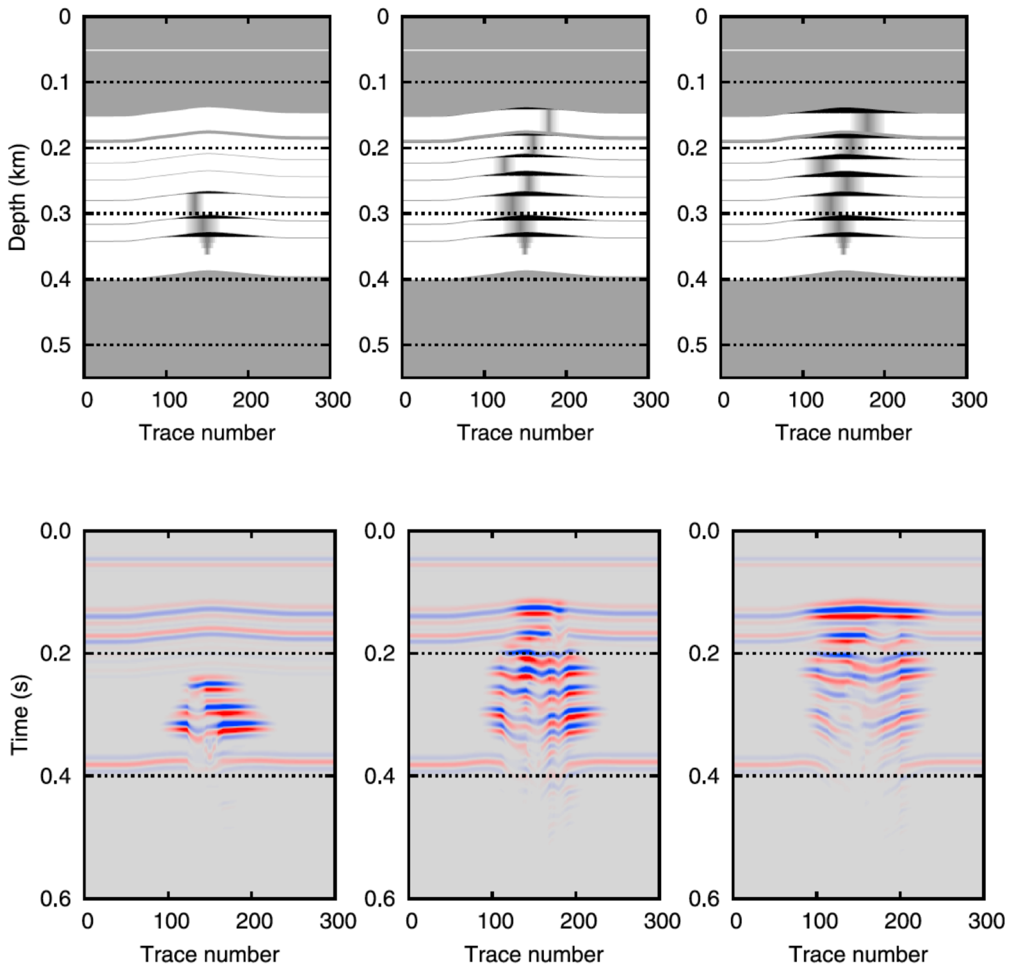
CO_2 geosequestration



Partially saturated formations are of interest in a wide variety of scientific scenarios. Seismic methods have a great potential to help in the remote characterization of such environments.

Seismic methods for fluid detection





Elastodynamics:

Hooke's law:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl},$$

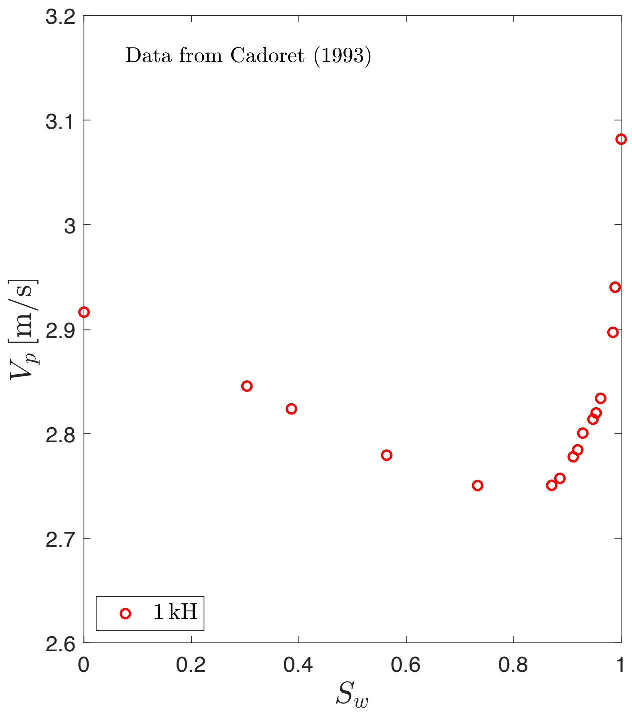
Equilibrium equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{d^2 u_i}{dt^2}.$$

$$V_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}, \quad V_s = \sqrt{\frac{\mu}{\rho}}.$$

Taken from Rubino et al (2011)

Fluid effects on seismic velocities



P-wave velocity as a function of saturation for a carbonate simple of 0.3 porosity for two measuring frequencies.

$$V_s(S_w) = \sqrt{\frac{\mu(S_w)}{\rho_b(S_w)}},$$

$$V_p(S_w) = \sqrt{\frac{K(S_w) + 4/3\mu(S_w)}{\rho_b(S_w)}},$$

Moduli are saturation dependent:

$$\sigma_{ij} = C_{ijkl}(S_w)\epsilon_{kl}$$

Saturation- and frequency-dependent effects

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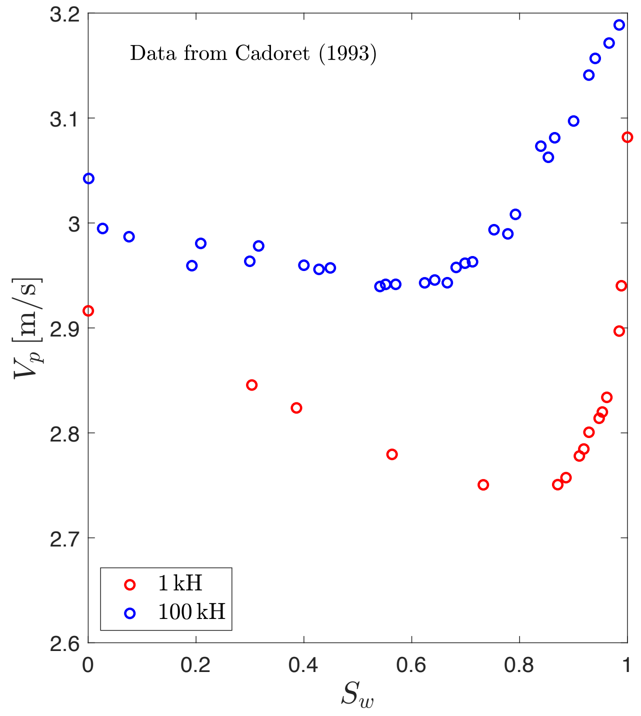


P-wave velocity as a function of saturation for a carbonate simple of 0.3 porosity for two measuring frequencies.

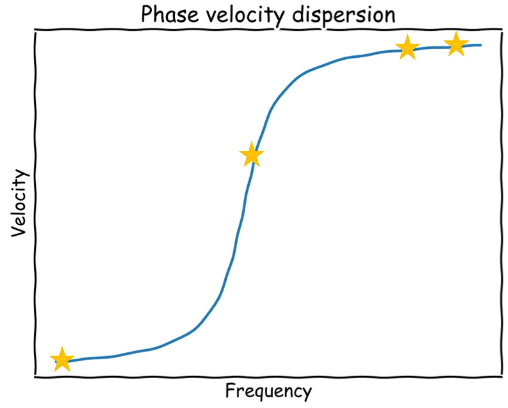
Seismic velocities are frequency-dependent:

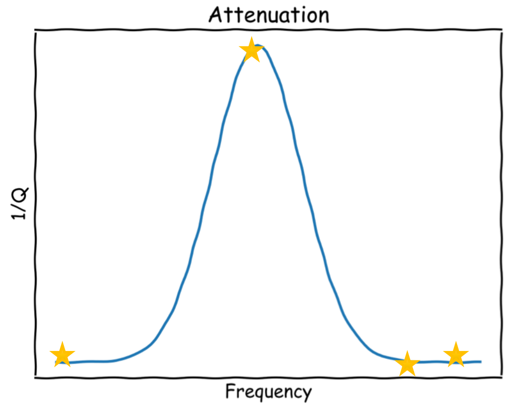
$$V_s(\omega, S_w) \qquad V_p(\omega, S_w) \quad \boxed{???}$$

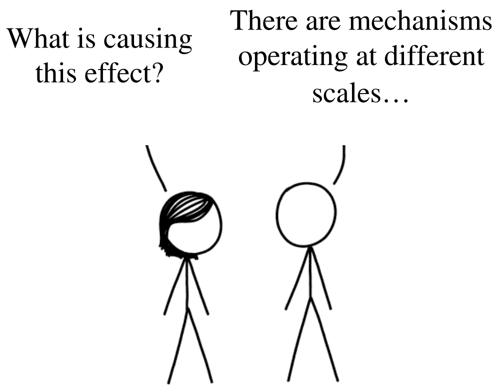
Implying that medium is effectively viscoelastic. Moduli are therefore complex-valued and frequency dependent, and Hooke's law holds in the space-frequency domain:

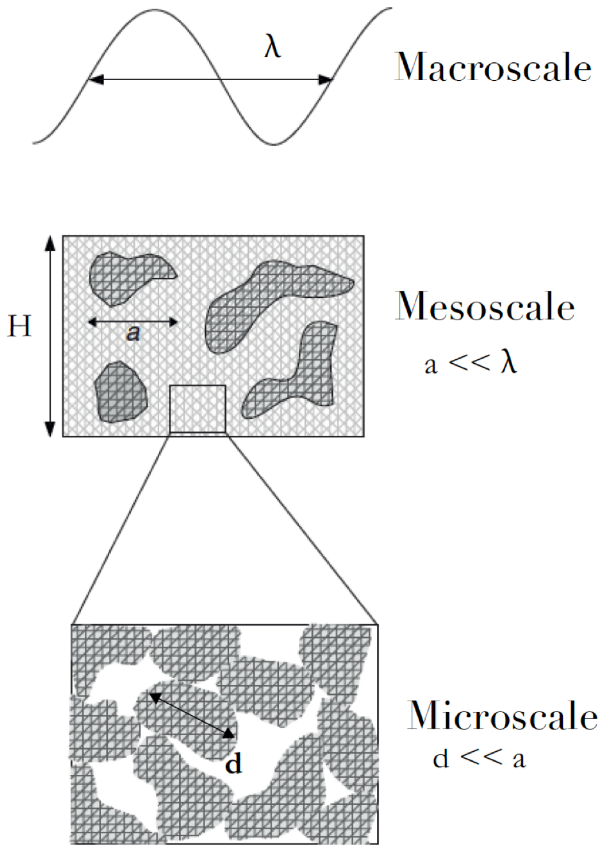
$$\hat{\sigma}_{ij}(\omega) = \hat{C}_{ijkl}(\omega, S_w)\hat{\epsilon}_{kl}(\omega)$$

Fluid coupling effects on seismic signatures









Fluid coupling effects on seismic signatures

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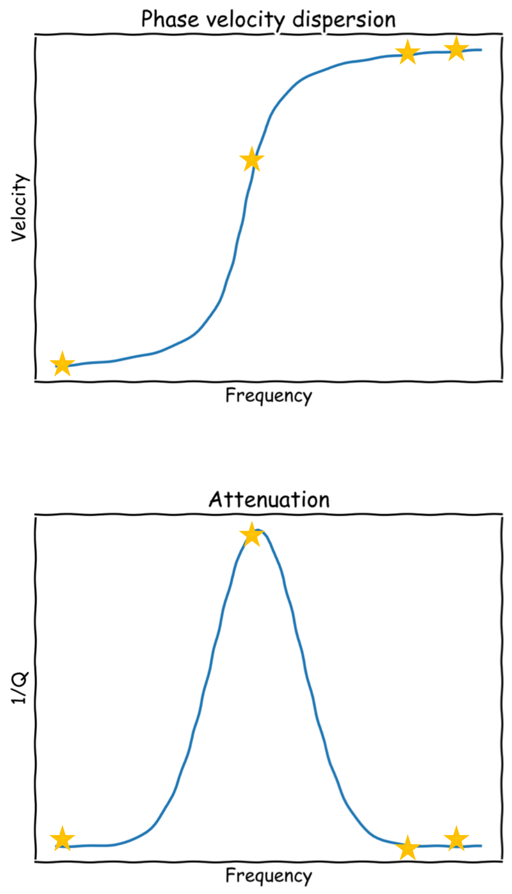
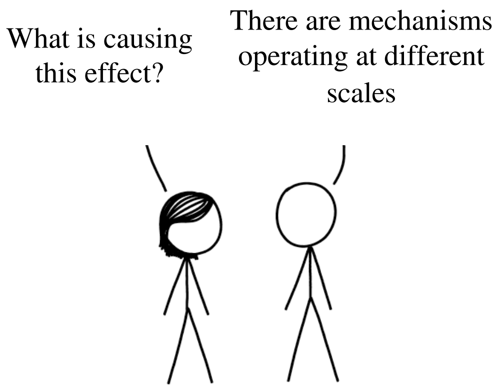
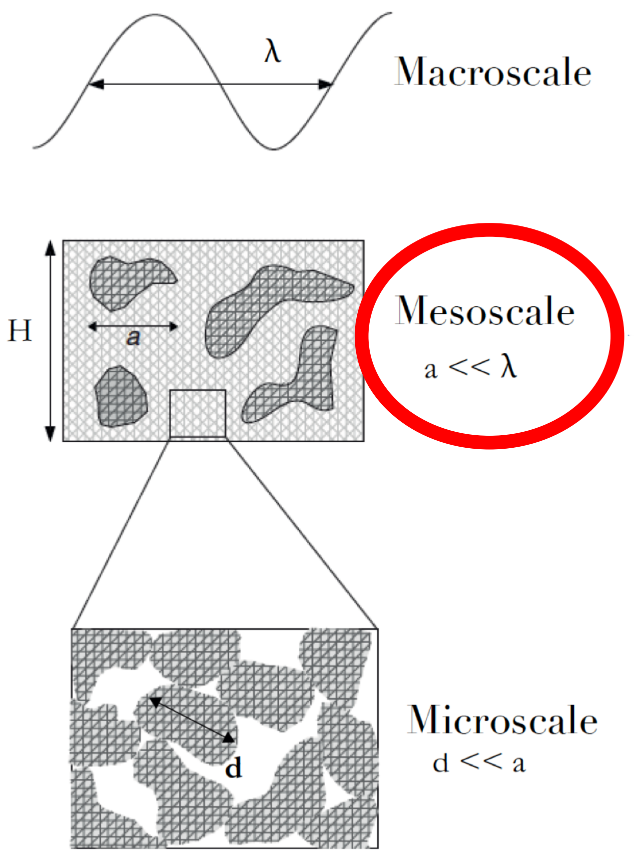
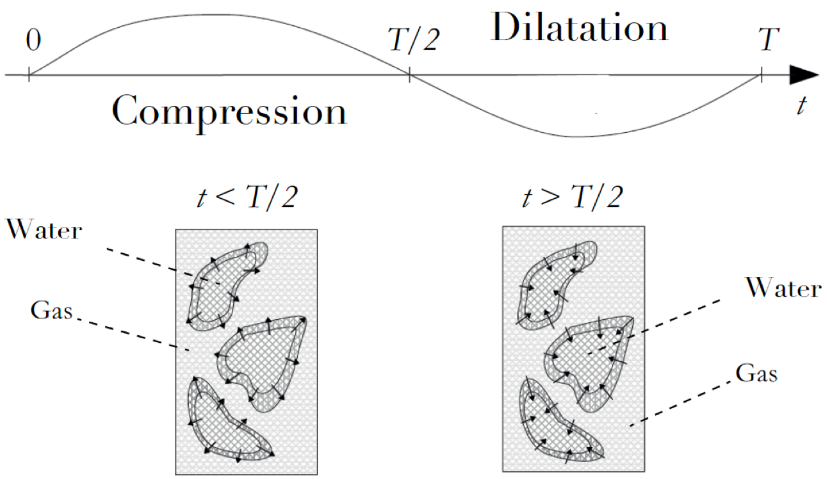


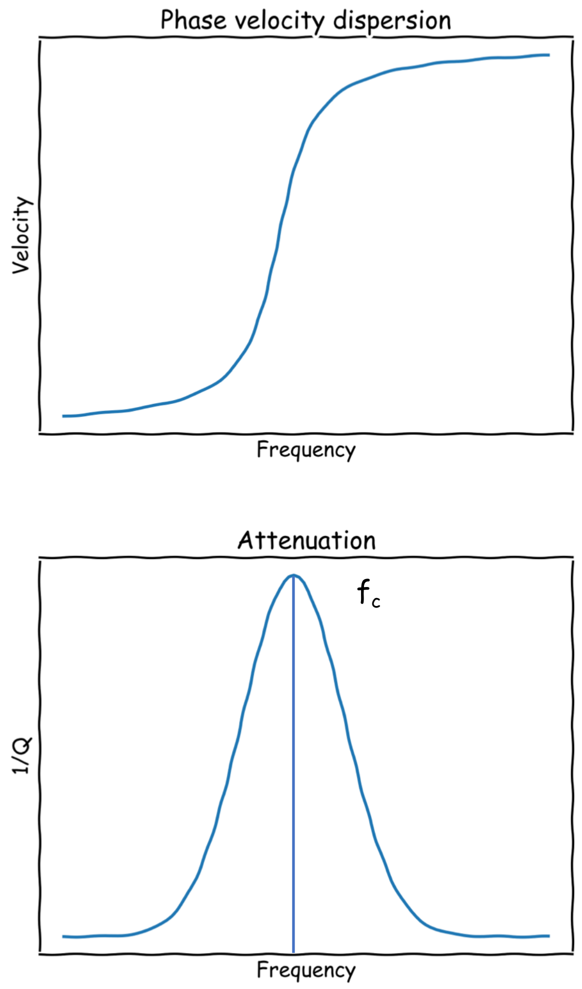




Wave-induced fluid flow



Modified from Müller *et al.* (2010).



The critical frequency is given by:

$$f_c \simeq \frac{D}{2\pi} \frac{1}{l_{meso}^2}$$

where D is the diffusivity of the medium, and l_{meso}^2 is the characteristic length of the heterogeneities.

Modelling wave-induced fluid flow effects

We solve Biot's (1941) poroelasticity equations in a representative elementary volume of the rock sample of interest under oscillatory forcing:

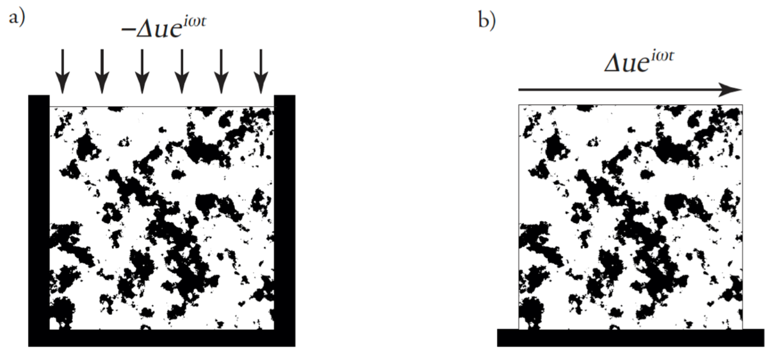


Illustration of (a) vertical, and (b) shear oscillatory relaxation tests to obtain the equivalent frequency-dependent moduli of the explored medium.

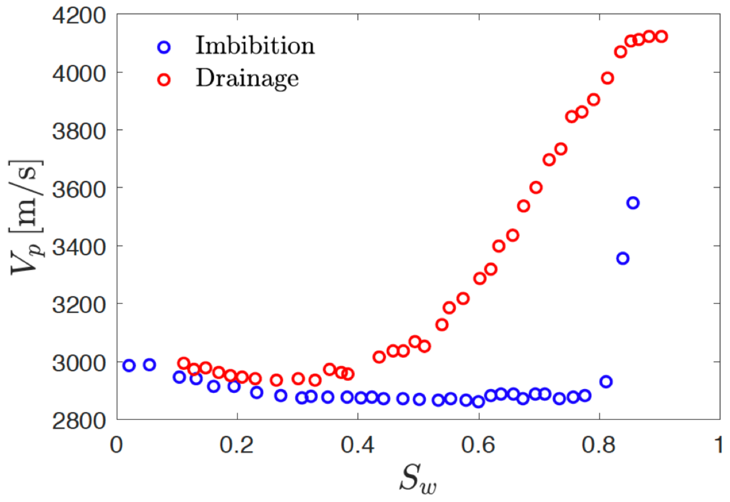
Plane-wave modulus

$$\boxed{M_c(\omega) = \frac{\langle \hat{\sigma}_{yy}(\omega) \rangle}{\langle \hat{\varepsilon}_{yy}(\omega) \rangle}}$$

$$Q_p^{-1}(\omega) = \frac{\Im\{M_c(\omega)\}}{\Re\{M_c(\omega)\}},$$

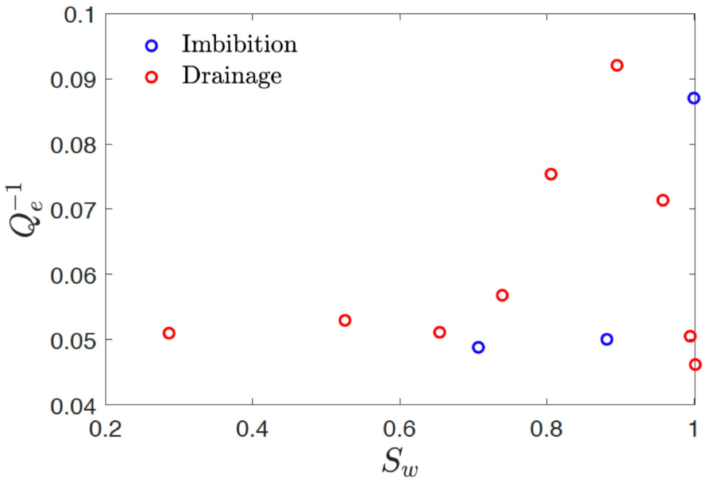
$$V_p(\omega) = \left[\Re\left\{\sqrt{\frac{\langle \rho_b \rangle}{M_c(\omega)}}\right\}\right]^{-1}.$$

Saturation hysteresis and seismic signatures

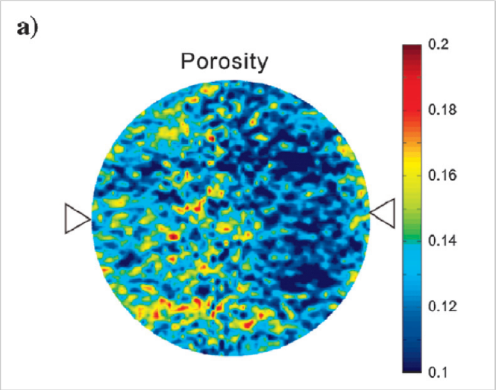


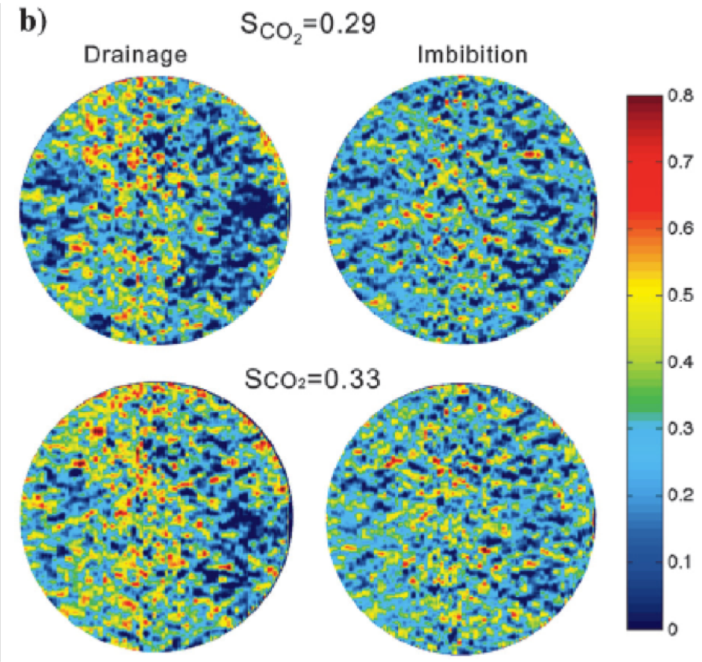
P wave velocity vs. water saturation in a Berea sandstone (Knight & Nolen-Hoeksema, 1992)

Extensional attenuation vs. water saturation in a partially saturated sandstone (Yin et al., 1992)



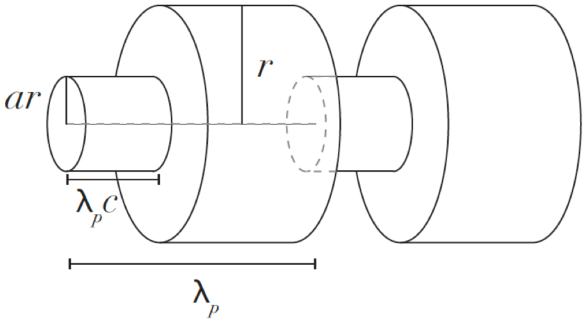
Saturation hysteresis and seismic signatures





(a) Porosity map and (b) distribution of CO_2 and water during drainage and imbibition (Zhang *et al.*, 2015).

Saturation hysteresis effects on the seismic signatures

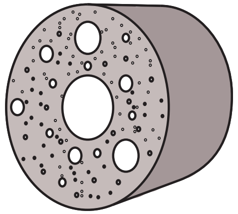


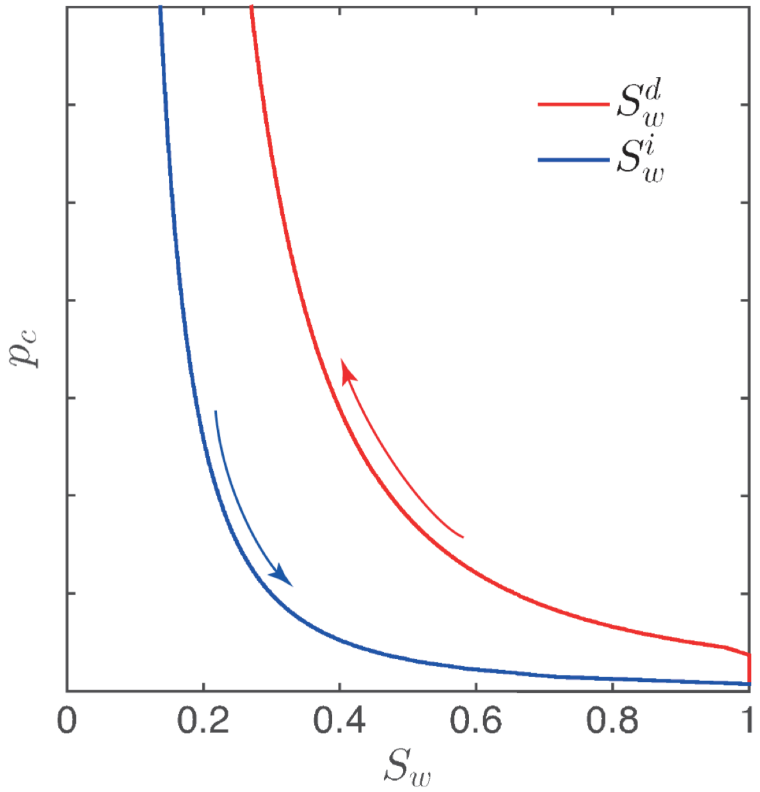
Constitutive model of constrictive capillary tubes with a fractal pore size distribution (Soldi et al., 2017).

ϕ

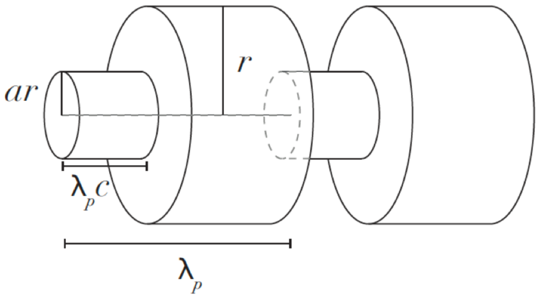
κ

$S_w^i(p_c), S_w^d(p_c)$





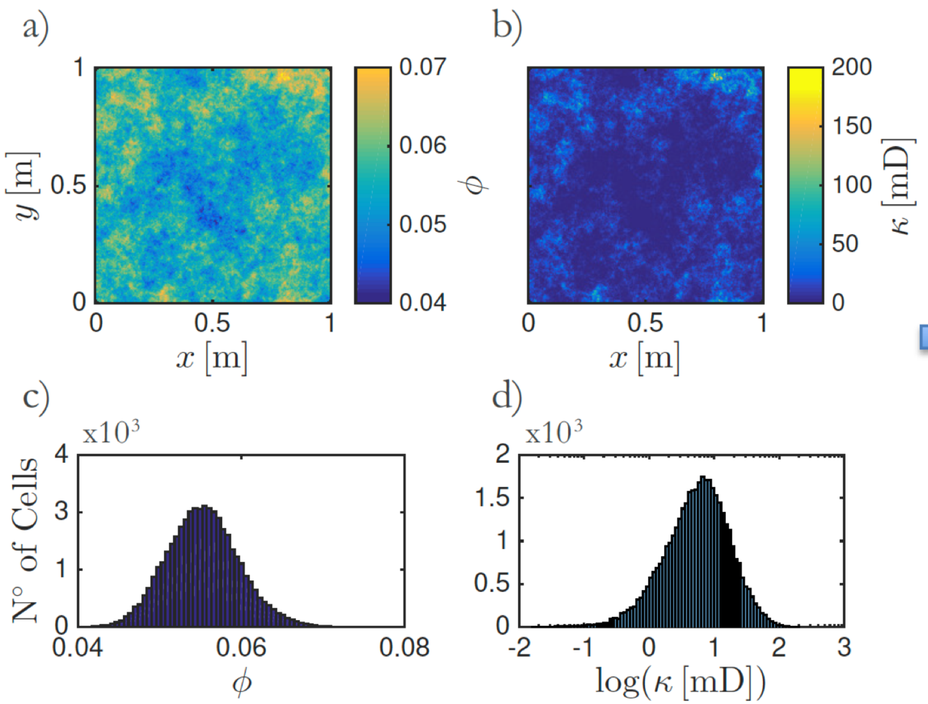
Saturation hysteresis effects on the seismic signatures

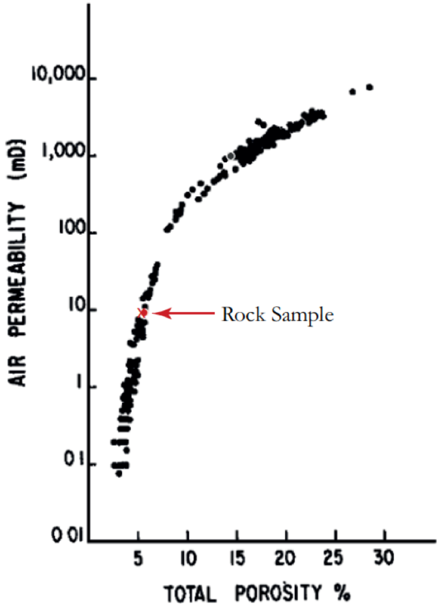


$$\phi_{ij}$$

$$\kappa_{ij}$$

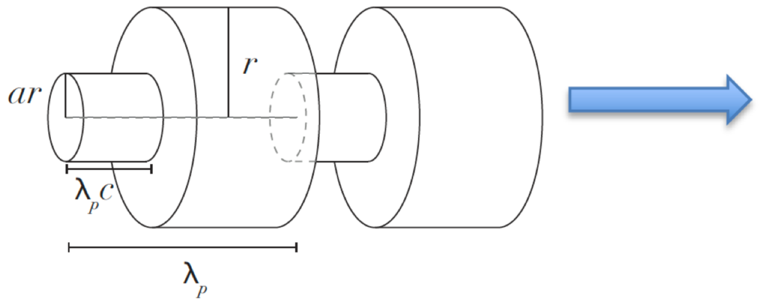
$$S^{i}_{wij}(p_c), S^{d}_{wij}(p_c)$$





Bourbié and Zinszner (1985)

Saturation hysteresis effects on the seismic signatures



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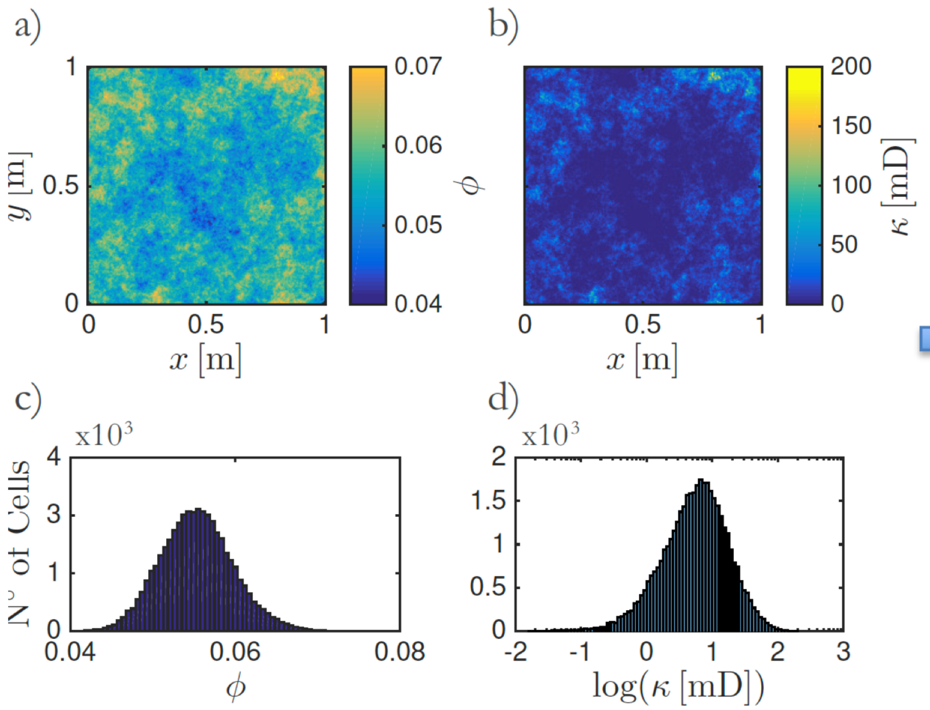
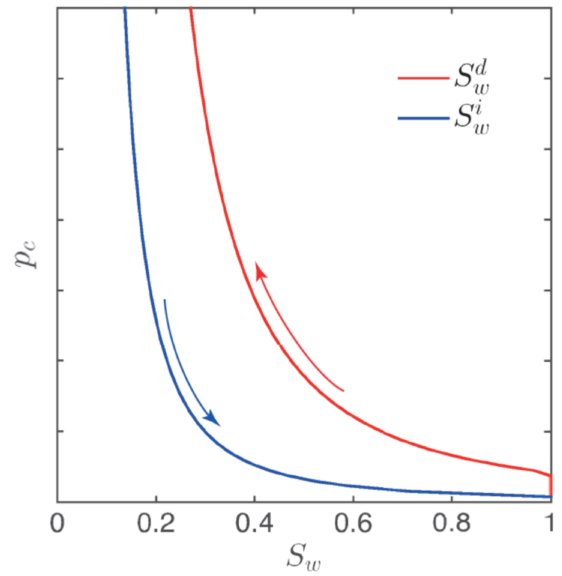
$$S^i_{wij}(p_c), S^d_{wij}(p_c)$$





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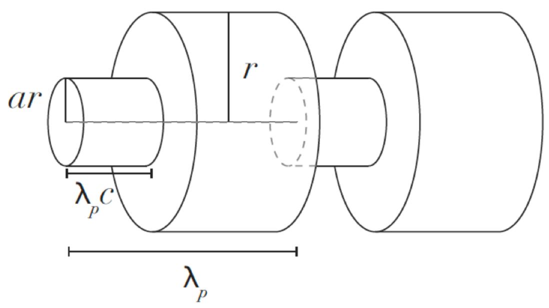
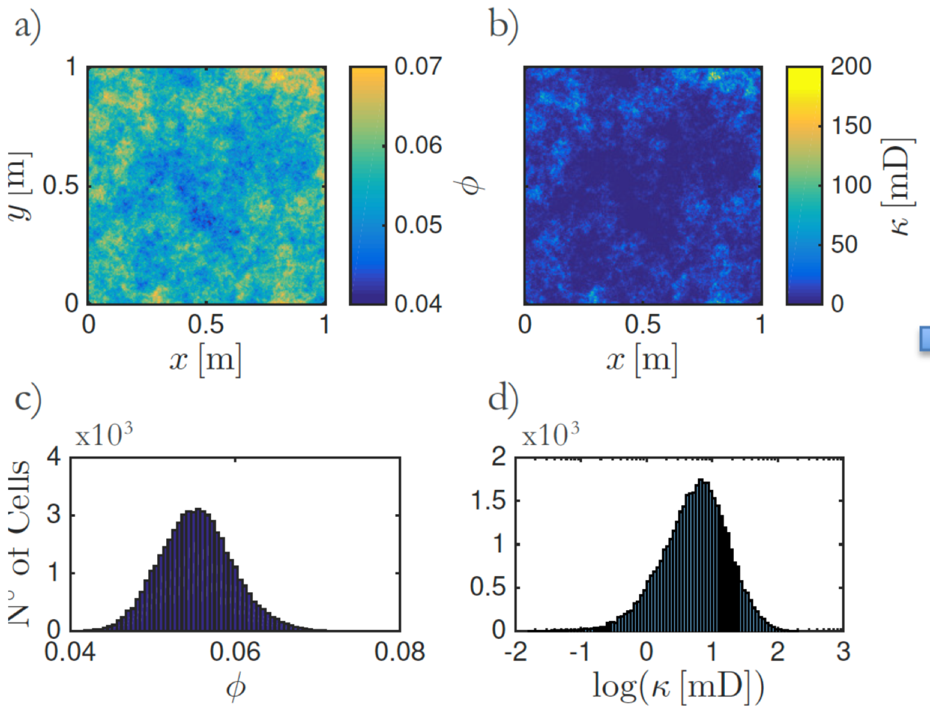
Saturation hysteresis effects on the seismic signatures

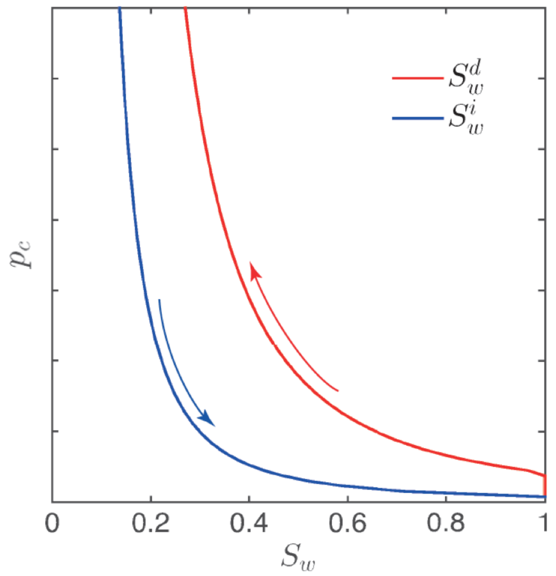


ϕ_{ij}

κ_{ij}

$S^i_{wij}(p_c), S^d_{wij}(p_c)$





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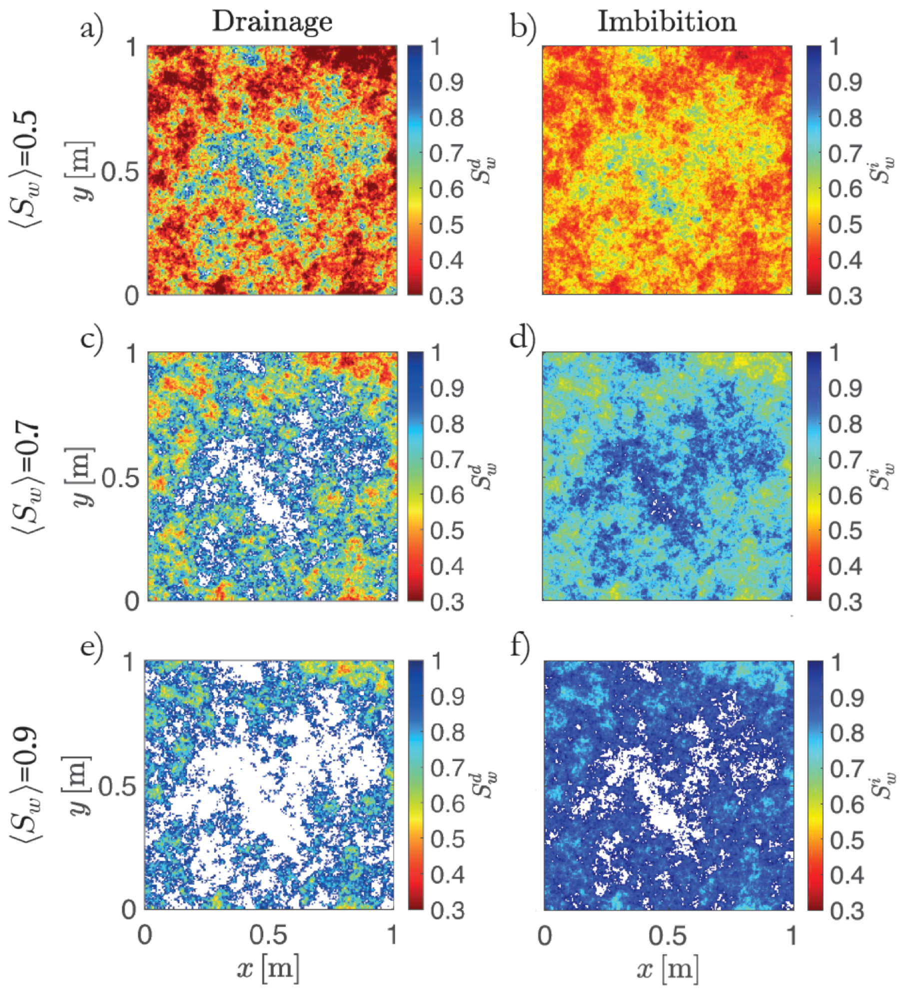
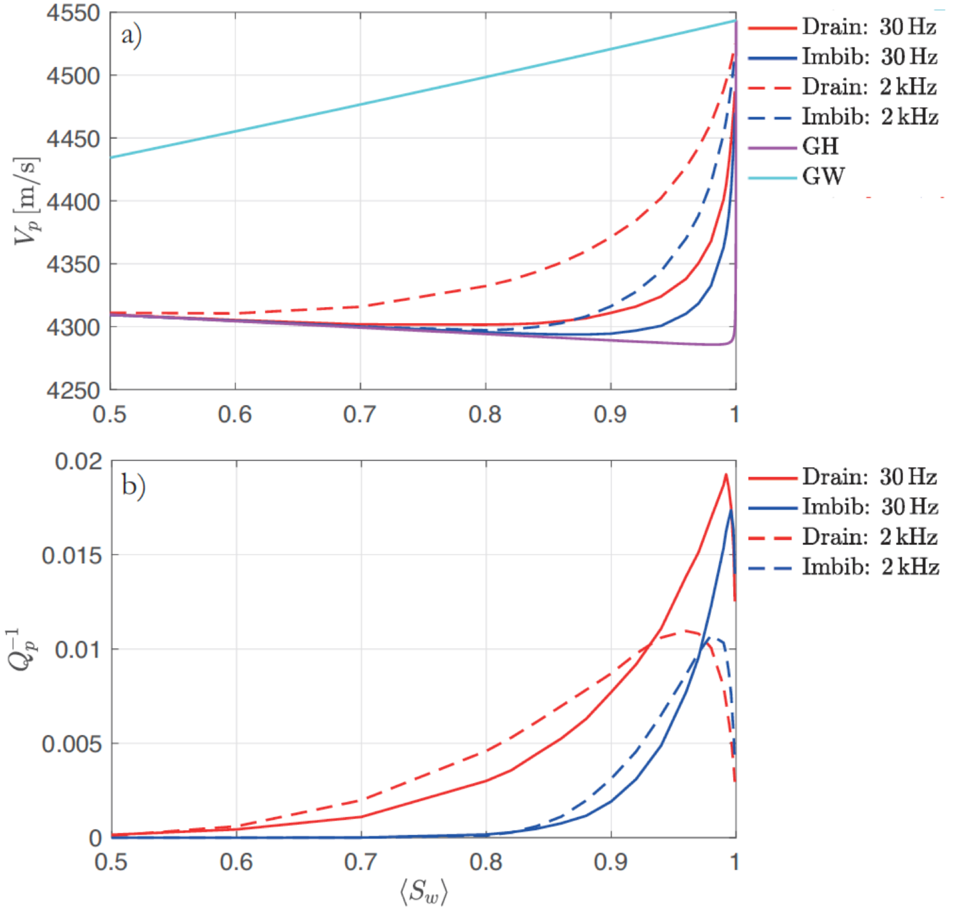
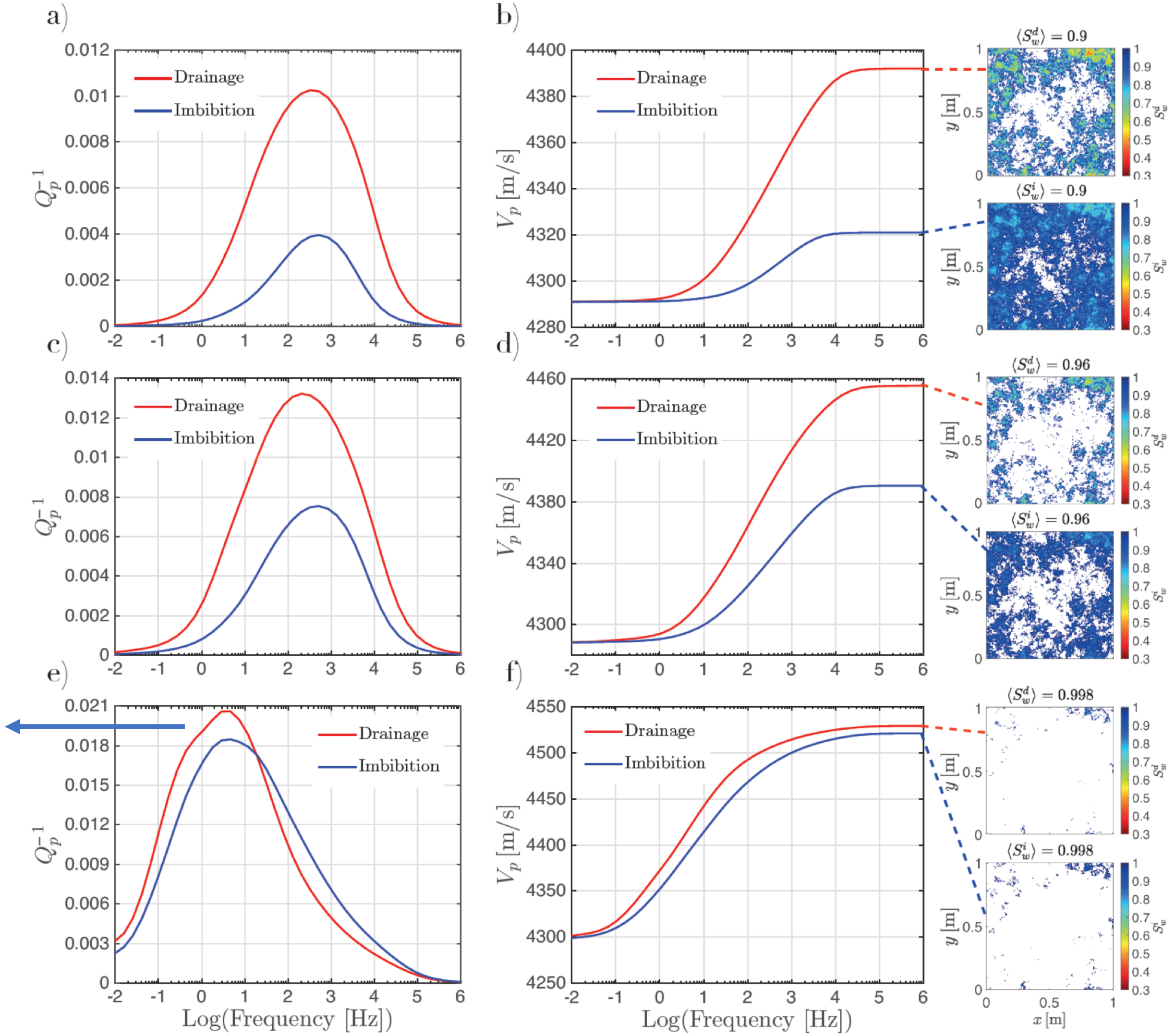
Saturation hysteresis effects on the seismic signatures





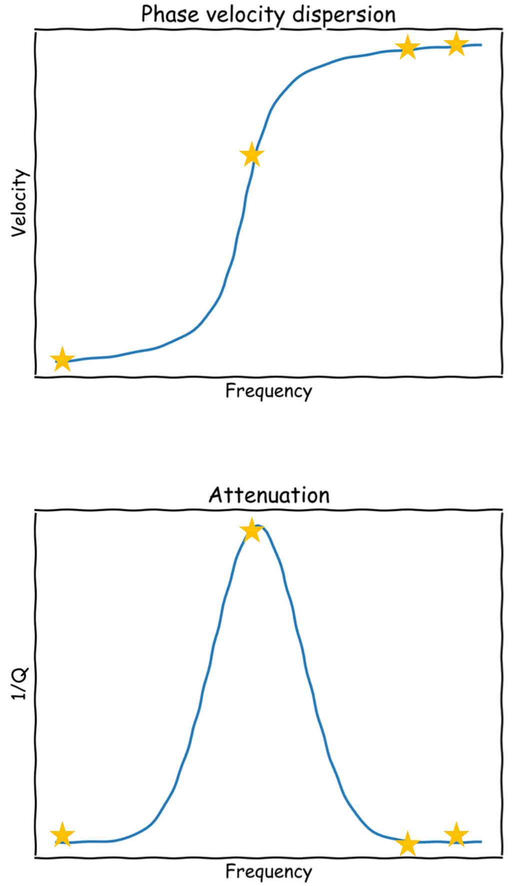
Taken from Solazzi et al (2019)

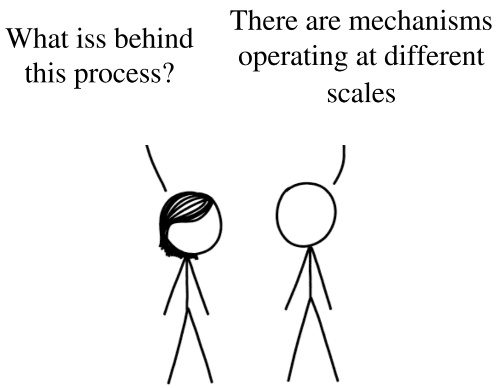
Saturation hysteresis effects on the seismic signatures

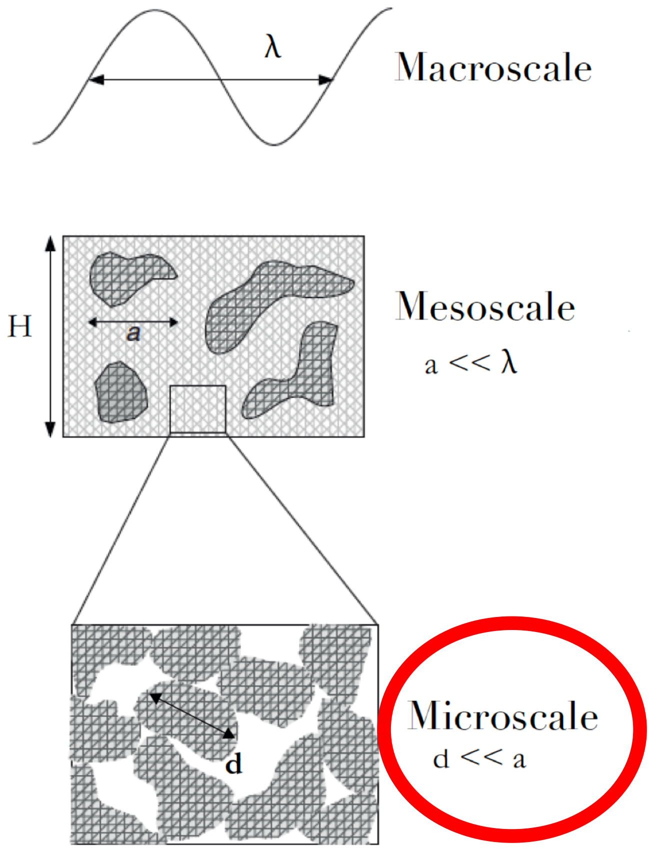


Taken from Solazzi et al (2019)

Fluid coupling effects on seismic waves

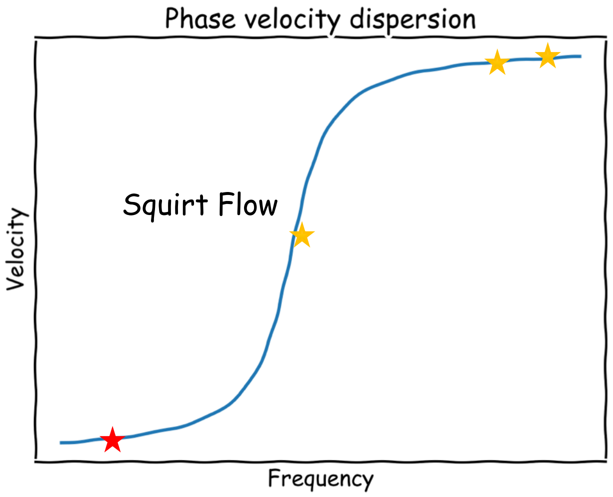


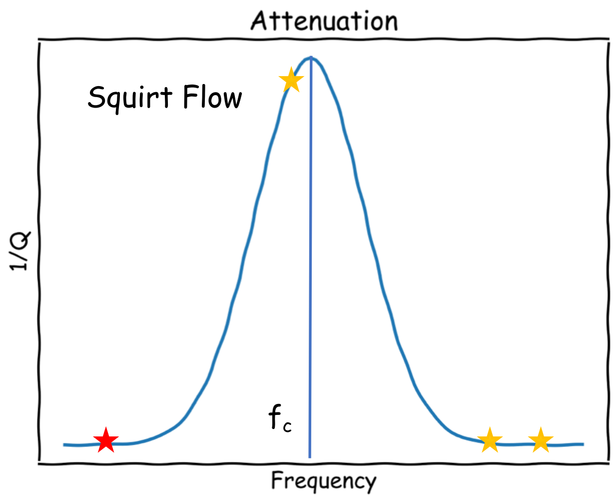


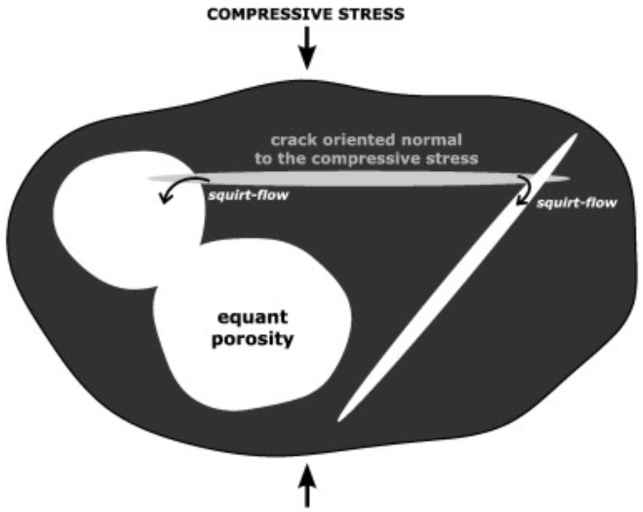


Taken from Muller et al (2010)

Squirt flow effects







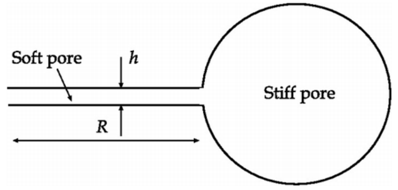
Squirt flow mechanism:

- Microcracks can be present in grains.
- Fluid presure diffusion occurs between these cracks and the connected pore space.

For most sedimentary water-saturated rocks this process occurs at frequencies larger 10^3 Hz.

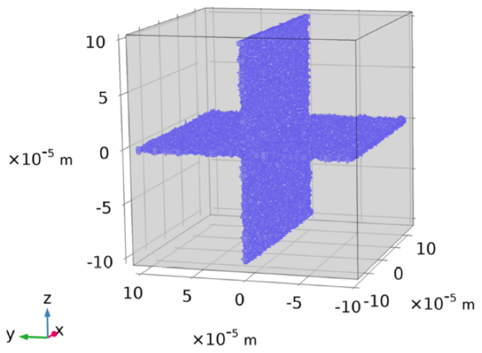
$$f_c \simeq \frac{1}{2}\frac{K_s}{\eta}\left(\frac{h}{L}\right)^3$$

Simple Geometries:



Taken from Gurevich et al (2009)

Numerical Modelling:



Taken from Lisa et al (2021)

Squirt flow effects on partially saturated cracks

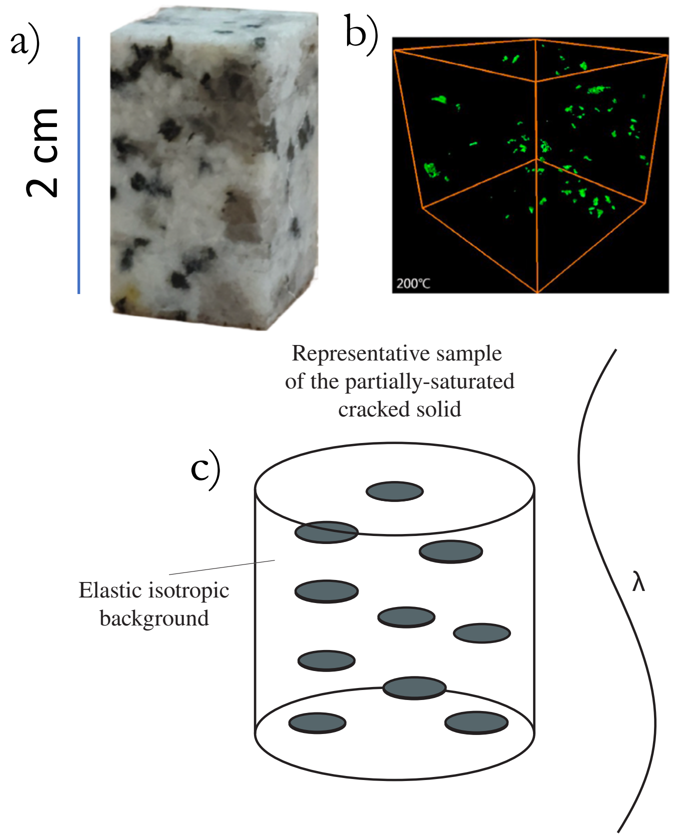


Fig 1: (a) Granite sample and (b) 3D CT-scan showing cracks in green (Fan et al., 2018). (c) Conceptual model.

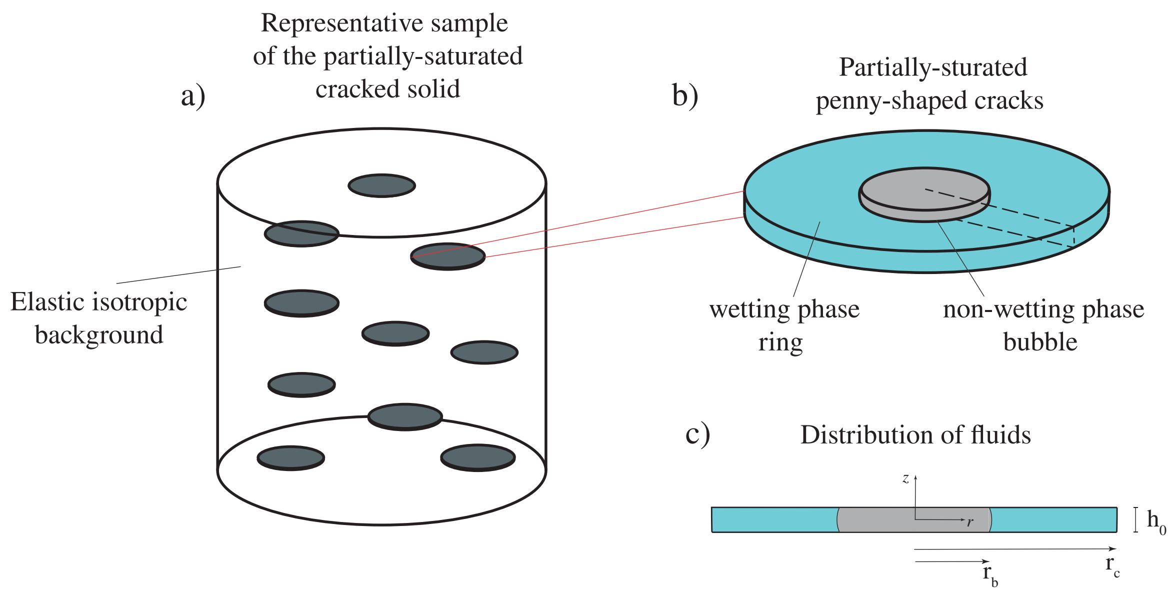


Fig 2: (a) Sketch of the cracked rock and (b) blowup of a partially saturated crack. (c) Vertical profile, illustrating the distribution of the fluid phases within the crack.

Taken from Solazzi et al (2021)

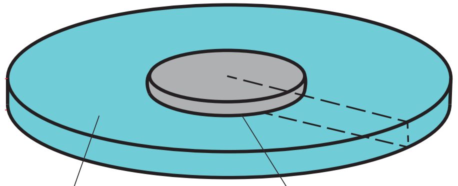
Squirt flow effects on partially saturated cracks

Cracks behave as if saturated with a frequency-dependent effective fluid with a bulk modulus

$$K_f^*(\omega) = S_\mathrm{w} K_\mathrm{w} \mathcal{T}_\mathrm{w}(\omega) + (1 - S_\mathrm{w}) K_\mathrm{n} \mathcal{T}_\mathrm{n}(\omega)$$

$\mathcal{T}_\mathrm{n}(\omega)$ and $\mathcal{T}_\mathrm{w}(\omega)$:e a combination of Bessel functions that include the crack geometrical properties, fluid compressibilities K_w and K_n, and saturation S_w.

Partially-sturated penny-shaped cracks



Modulus dispersion and attenuation as functions of saturation

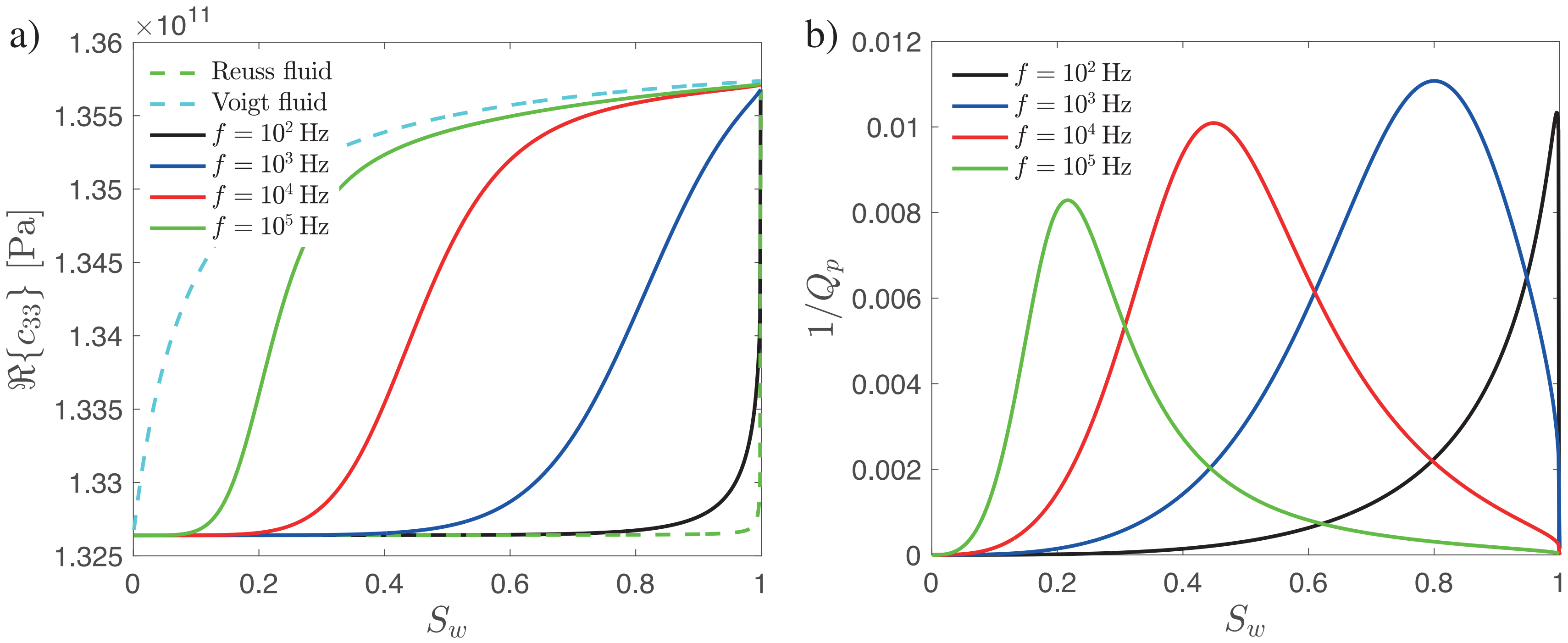
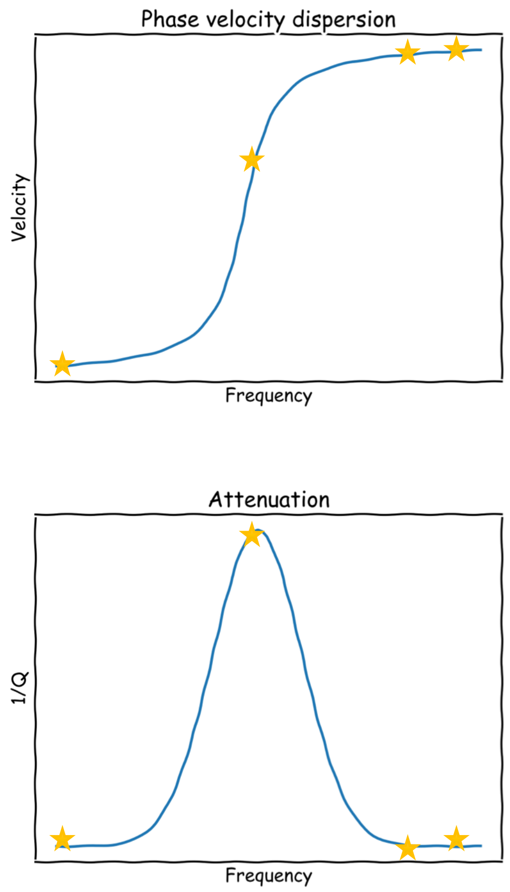


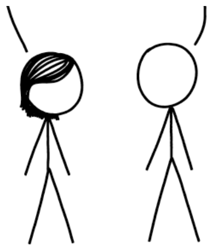
Fig 5: (a) Plane wave modulus $\Re\{c_{33}\}$ and (b) inverse quality factor Q_p^{-1} as functions of saturation for vertically travelling P-waves.

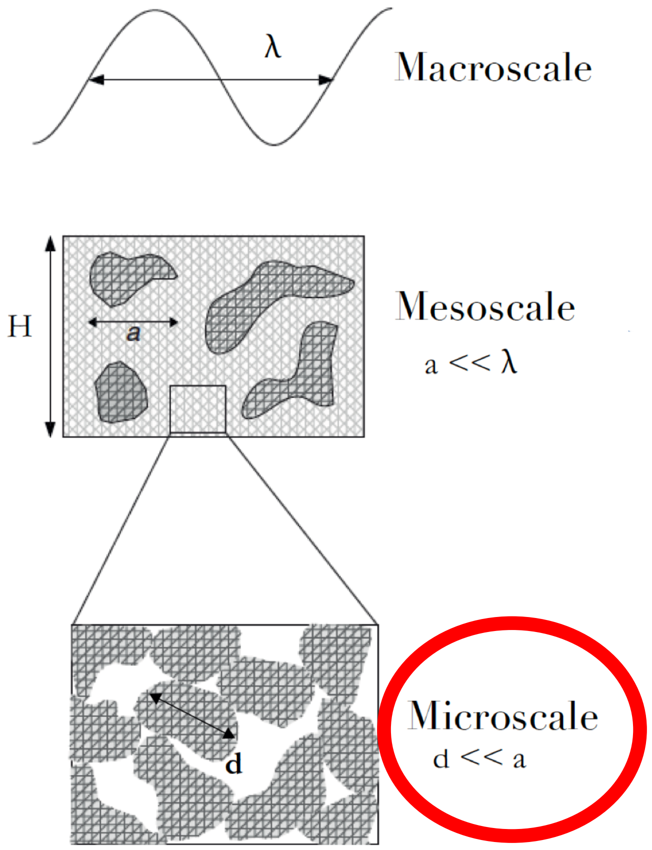
Fluid coupling effects on seismic signatures



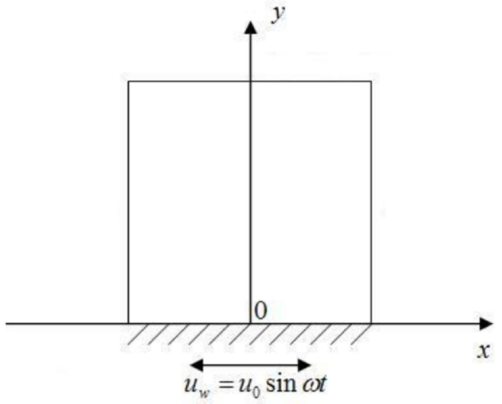
What iss behind this process?

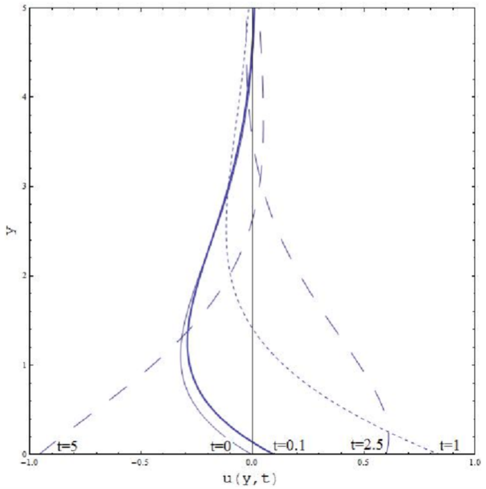
There are mechanisms operating at different scales

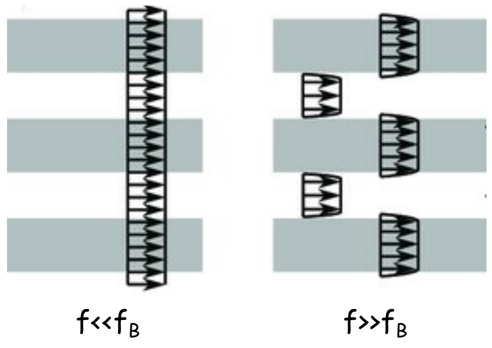




Biot’s intrinsic mechanism





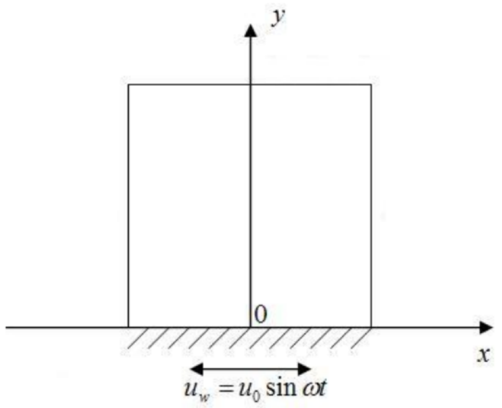


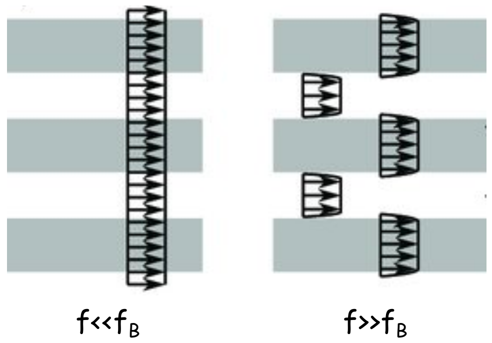
<u>Biot’s intrinsic mechanism:</u>

- Acceleration exherted by the passing wave.
- Flow is in the vicosity-dominated regime (Poiseuille flow) for $f<<f_B$.
- For much higher frequencies, the flow is inertia-dominated.

For most sedimentary water-saturated rocks this **normally process occurs at frequencies larger than 10^4 Hz**. Dominated by permeability and fluid viscoity.

Biot's intrinsic mechanism





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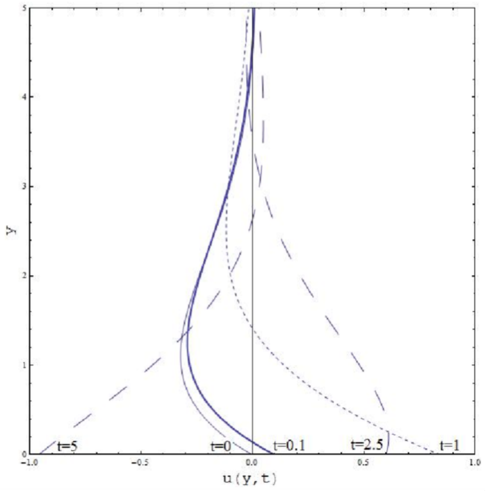
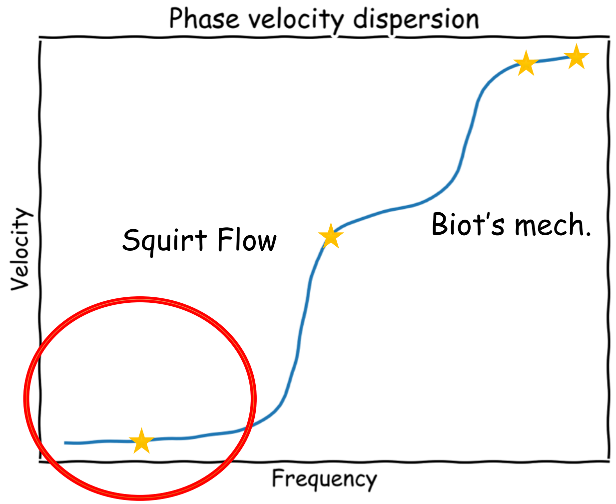
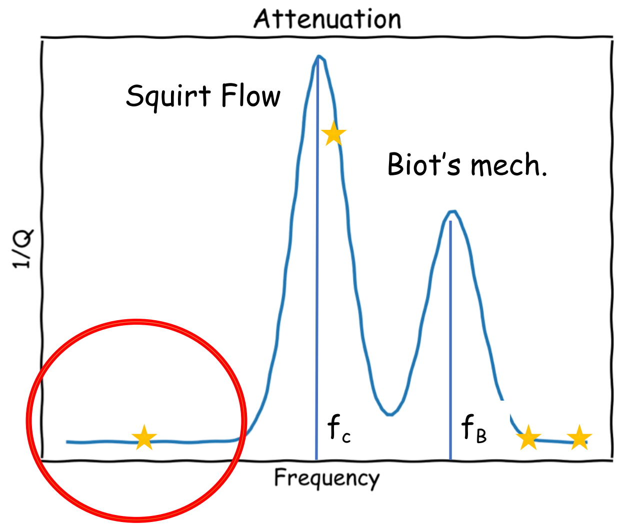
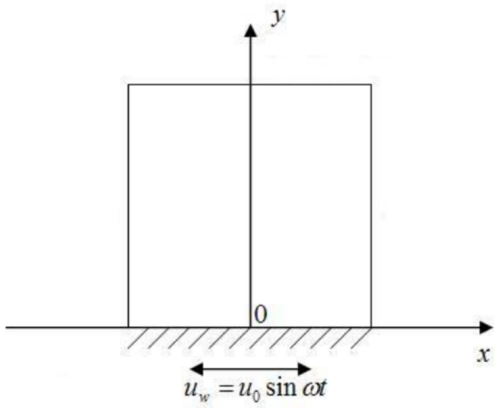
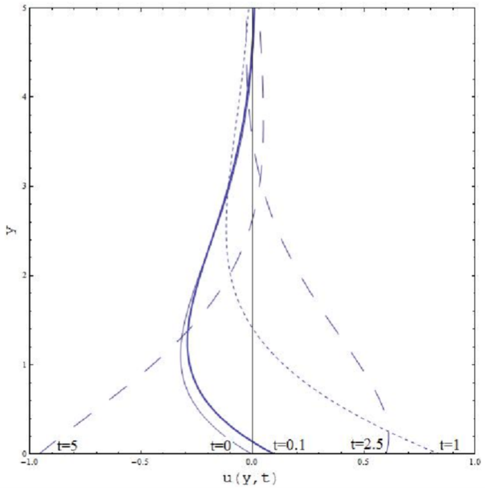






Biot’s intrinsic mechanism: Dynamic permeability



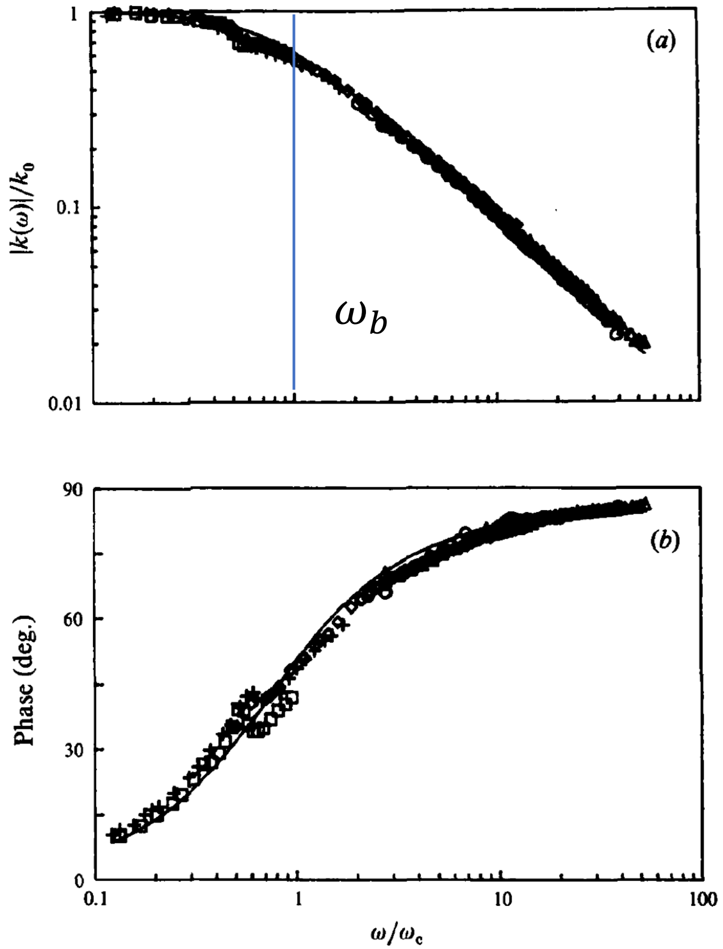


The classic formulation of Darcy’s law only holds for viscous dominated flow:

$$w = -\frac{\kappa_0}{\eta} \nabla p_f$$

If we want to use it for inertial flow, we need a frequency-dependent and complex-valued permeability:

$$\widehat{w}(\omega) = -\frac{\hat{\kappa}(\omega)}{\eta} \nabla \hat{p}_f(\omega)$$

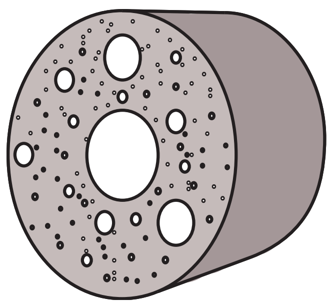


Data from Smeulders et al (1992)

On the dynamic permeability of partially saturated porous media

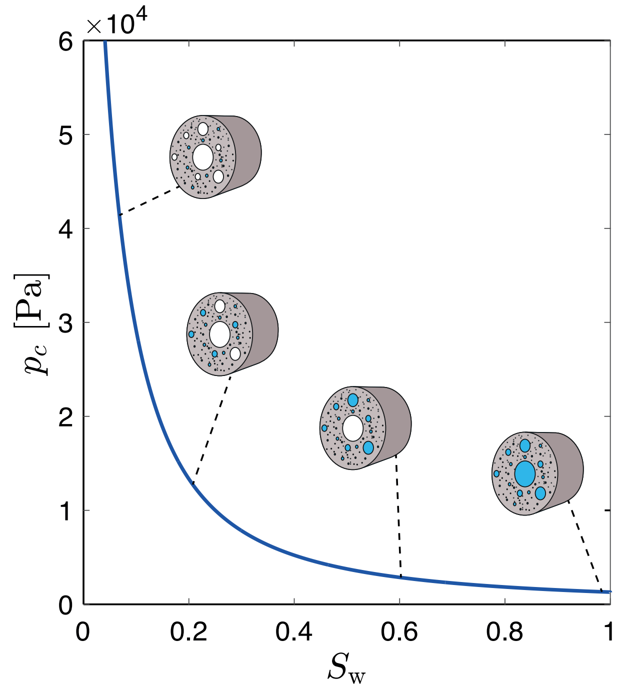
While the frequency-dependence of permeability under fully saturated conditions has been studied for decades, the corresponding characteristics of <u>partially saturated porous media</u> remain unexplored.

We conceptualize the pore space as a bundle of tubes with a known pore size distribution

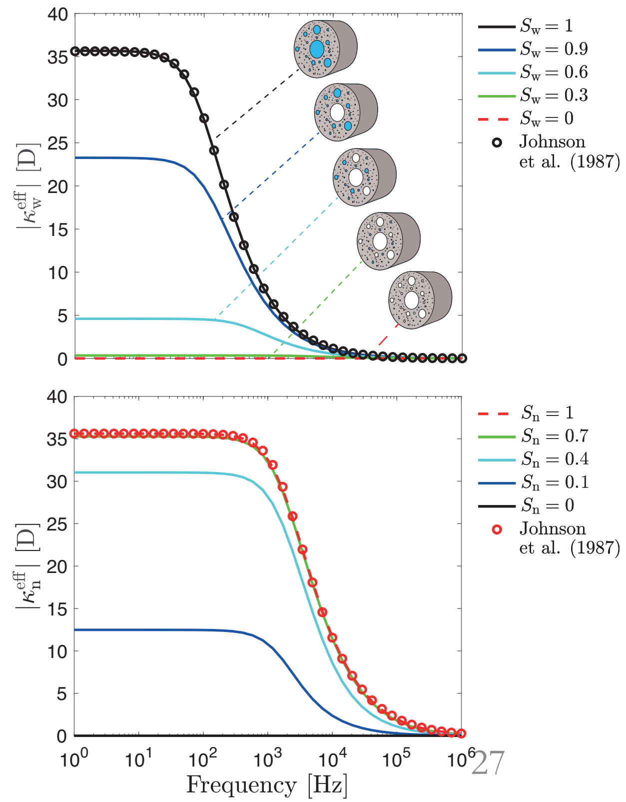


Taken from Solazzi et al (2020)

We introduce a capillary pressure-saturation relationship to saturate the medium

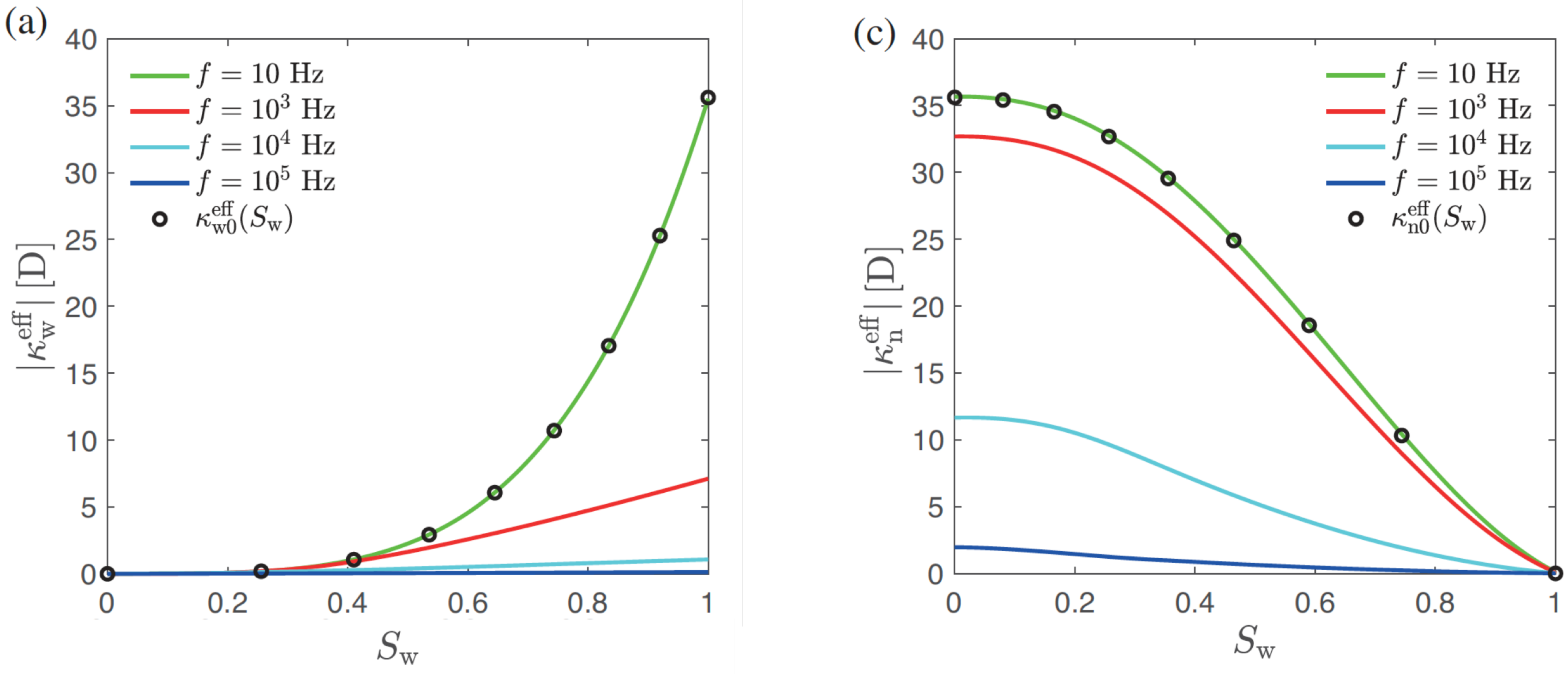


Frequency and saturation dependent permeability estimates



On the dynamic permeability of partially saturated porous media

Effective permeability functions, as sensed by seismic waves in oscillatory flow, are functions of the saturation and of the frequency



Taken from Solazzi et al (2020)

Conclusions

- There are several mechanisms that induce frequency dependence in the effective elastic moduli of fluid saturated porous media.

- Wave-induced fluid flow in the mesoscopic scale is a fluid pressure diffusion process. It is modelled using Biot's poroelasticity equations. Elastic moduli not only depend on the saturation state but, also, on the geometry of the fluid distributions.

- Squirt flow effects occur due to fluid pressure diffusion at the microscopic scale. It is modelled solving Navier-Stokes equations coupled with the elasticity equations. Elastic moduli depend on the geometry of the pores, fluid characteristic, and saturation state.

- Biot's intrinsic mechanism is associated with fluid drag produced by an accelerated pore matrix. The effective permeabilities sensed by the wave in partially saturated conditions are saturation and frequency dependent.

- Understanding the physical reasons behind these processes may permit a better characterization of the hydraulic and mechanical properties of the explored media using seismic methods.

CFMR
COMITÉ FRANÇAIS DE MÉCANIQUE DES ROCHES

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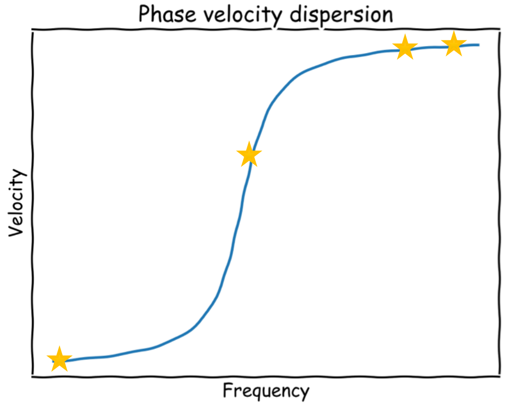
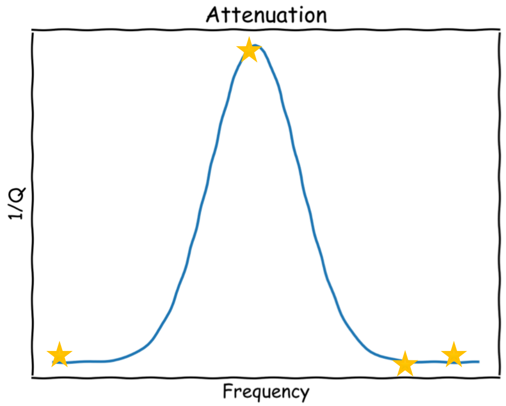
Faculté des géosciences et de l'environnement

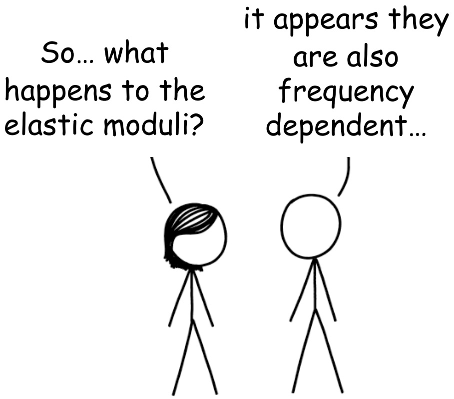
Thank you very much!

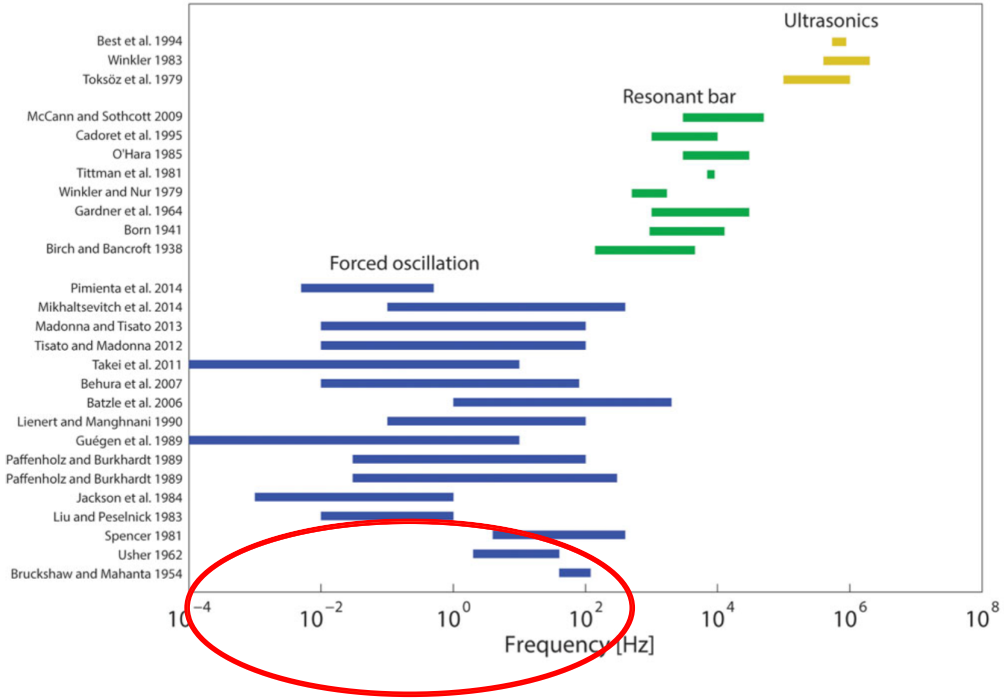
This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sktodowska-Curie grant agreement No 101007851

Fluid coupling effects on seismic signatures

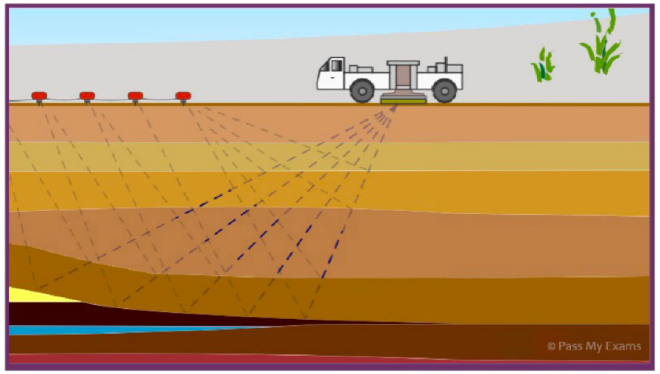








Seismic Method



Elastodynamics:

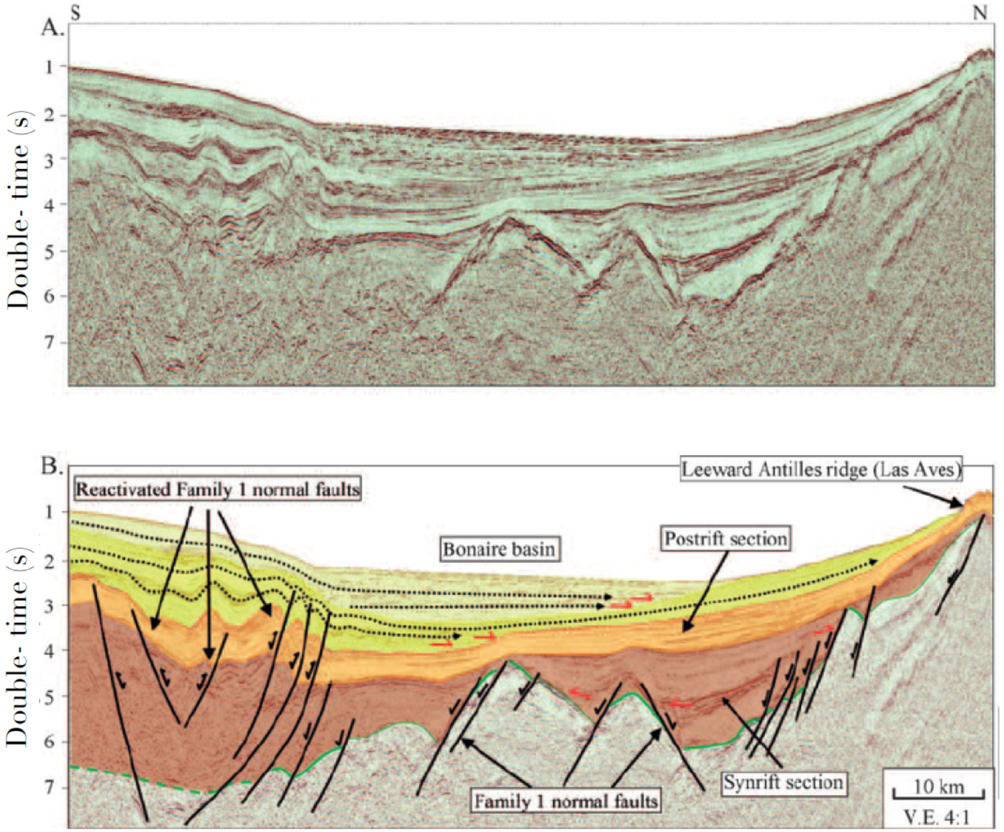
Hooke's law:

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl},$$

Equilibrium equation:

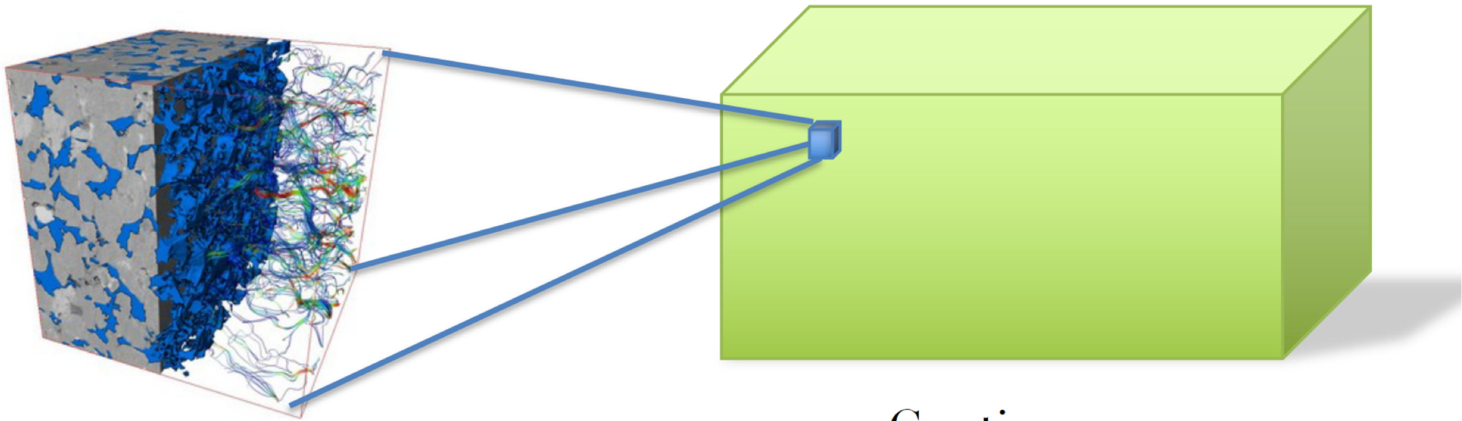
$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{d^2 u_i}{dt^2}.$$

$$V_p = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}}, \quad V_s = \sqrt{\frac{\mu}{\rho}}.$$



Gorney *et al.* (2007).

Biot's theory of poroelasticity



Representative Elementary Volume

Continuum Medium

<u>Variables of Biot's theory:</u>

u : Solid displacement.

w : Relative fluid displacement

$\zeta = -\nabla . w$: Variation in the fluid content

p_f :Fluid pressure

σ : Total stress tensor

ε : Strain tensor

Giorgiadis *et al.* (2013)

Biot's theory of poroelasticity

Constitutive relations

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}(\lambda_c\varepsilon_{kk} - \alpha M\zeta),$$
$$p_f = -\alpha M\varepsilon_{kk} + M\zeta.$$

Coefficients

$$\left\{\begin{array}{ll} \alpha & \mu \\ M & \lambda_c \end{array}\right.$$

Biot's poroelastic equations:

$$\nabla.\sigma = \rho_b\ddot{u} + \rho_f\ddot{w},$$
$$-\nabla p_f = \rho_f\,\ddot{u} + g\ddot{w} + \frac{\eta}{\kappa}\dot{w}.$$

$\Rightarrow$ Supports 3 types of waves

P

P_2

S

Biot's theory of poroelasticity

<u>Constitutive relations</u>

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \delta_{ij}(\lambda_c\varepsilon_{kk} - \alpha M\zeta),$$
$$p_f = -\alpha M\varepsilon_{kk} + M\zeta.$$

Coefficients

$$\alpha \quad \mu$$
$$M \quad \lambda_c$$

<u>Biot's quasi-static equations:</u>

$$\nabla.\sigma = 0,$$
$$-\nabla p_f = \frac{\eta}{\kappa}\dot{w}.$$

Supports 3 types of waves

P
P_2
S

Equilibrium of forces and Darcy equation

Validation of the analytical solution

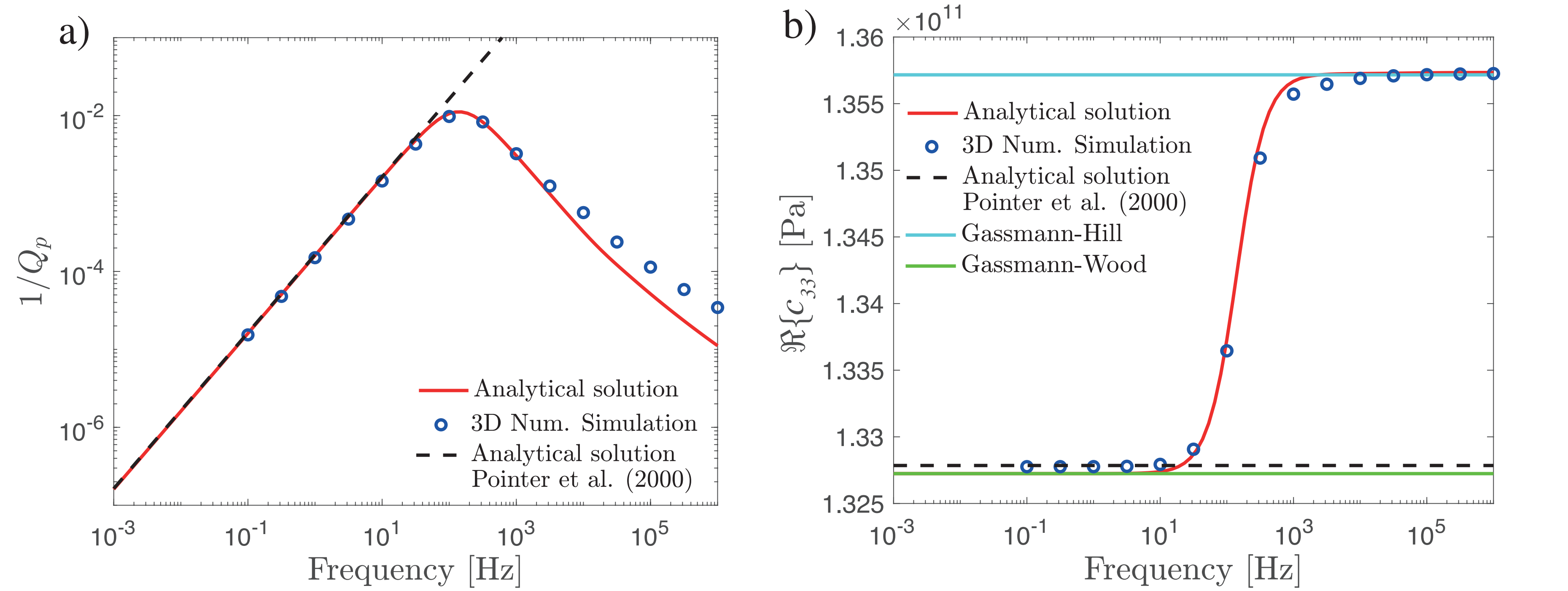
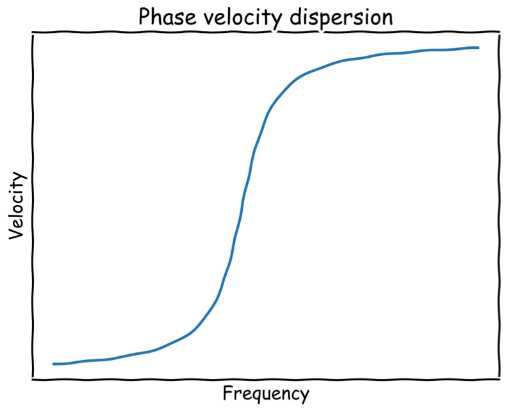
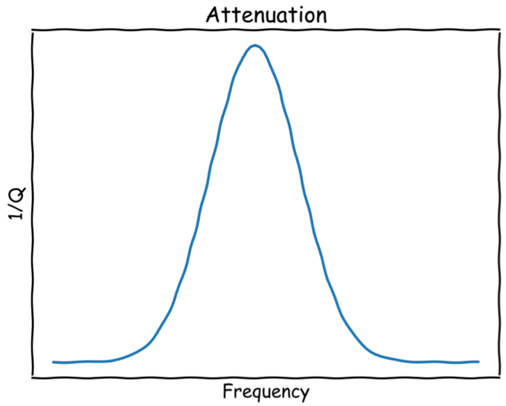


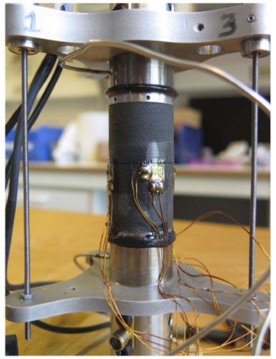
Fig 3: (a) Inverse quality factor Q_p^{-1} and (b) plane wave modulus $\Re\{c_{33}\}$ as functions of frequency for vertically travelling P-waves. Background properties are taken from a Westerly Granite characterized by a fracture density of $\varepsilon = 4.6\times10^{-3}$. Cracks have an aspect ratio $\alpha = 3.6\times10^{-3}$ and an aperture $h_0 = 10\ \mu$m. Fluids properties are those of glycerin and air.

Fluid coupling effects on seismic signatures

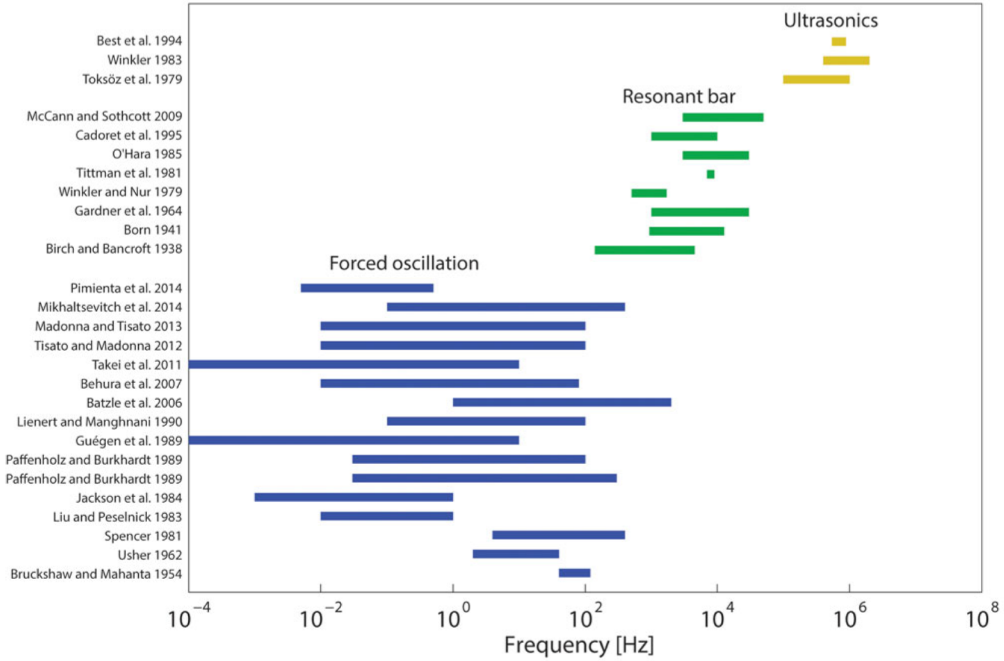




Do we also observe this in the lab?



Forced oscillation apparatus, SINTEF, Norway (Lozovyi et al, 2019)



Frequency ranges of different techniques to measure attenuation and dispersion of (Subramaniyan et al, 2014).

Gassmann’s equations

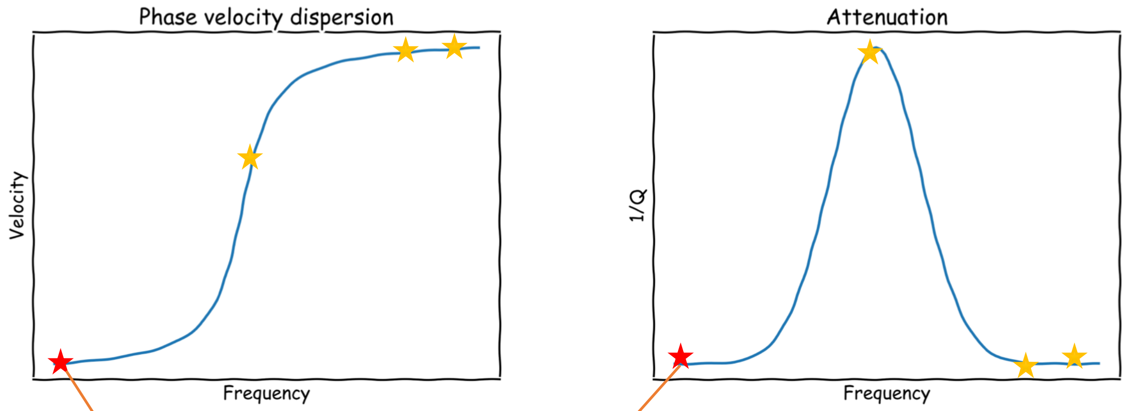
Gasssmann’s (1951) model:

$$K_{sat} = K_{dry} + \frac{\left(1 - K_{dry}/K_g\right)^2}{\frac{\phi}{K_f} + \frac{(1-\phi)}{K_g} + \frac{K_{dry}}{K_g^2}},$$

$$\mu_{sat} = \mu_{dry},$$

$$V_s(f) = \sqrt{\frac{\mu}{\rho_b}},$$

$$V_p(f) = \sqrt{\frac{K + 4/3\mu}{\rho_b}},$$



Gassmann’s (1951) low-frequency model

Fluid effects on seismic signatures

There are three main dissipation processes associated with fluid/solid coupling:

- Intrinsic Biot's mechanism
- Squirt flow mechanism

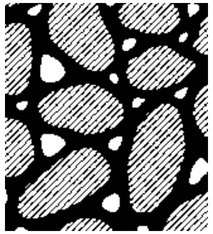
Take place in a homogeneous and fully-saturated porous media, <u>and are modified in presence of partial saturation</u>.

- Fluid pressure diffusion in the mesoscopic scale

Does not take place in homogeneous and fully-saturated media. However, it arises in partial saturation scenarios!

Fluid content and Seismic Signatures

Partially saturated medium



Water

Air

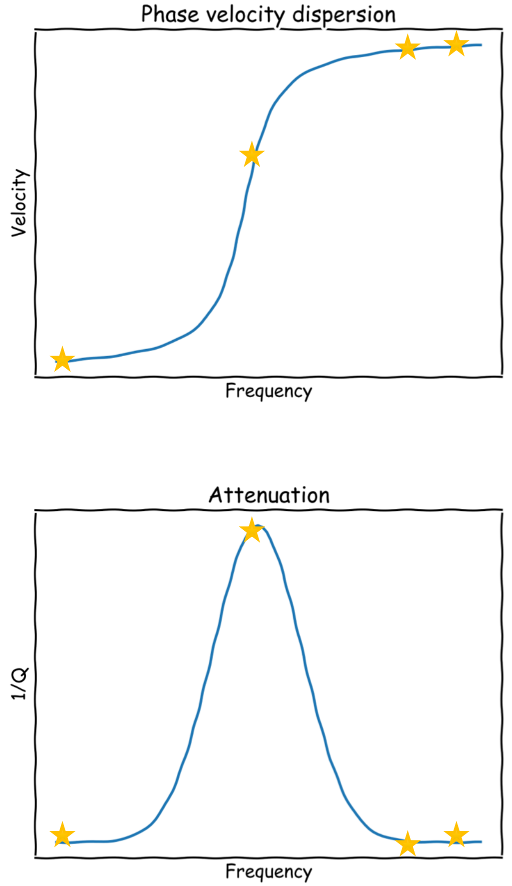
Solid Grains

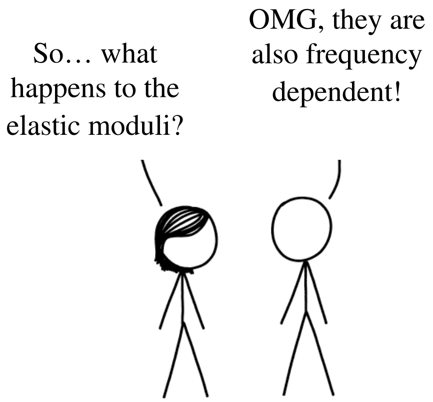
Mechanical properties

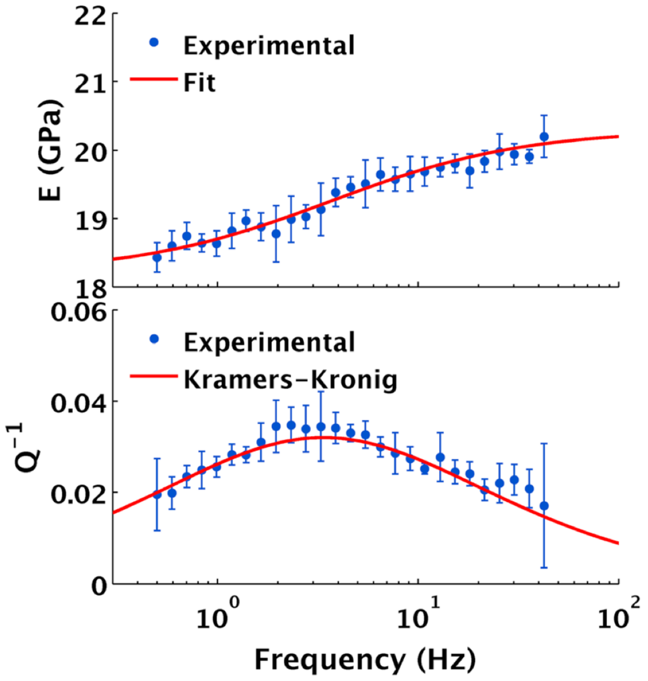
Seismic Signatures

Hydraulic properties

Fluid coupling effects on seismic signatures







Illustrative example of experimentally retrieved frequency-dependent Young modulus E and inverse quality factor 1/Q in a Berea sandstone with a saturation of 99% (Chapman et al, 2016).