Modelling the effective elastic moduli of partially saturated porous rocks

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Outline:

• Motivation

• Mesoscopic Wave-Induced Fluid Flow:
  ✓ Physical process
  ✓ Poroelastic modelling
  ✓ Case study: Hysteresis effects

• Squirt flow:
  ✓ Physical process
  ✓ Modelling approaches
  ✓ Case study: Partially saturated crack

• Biot’s intrinsic mechanism:
  ✓ Physical process
  ✓ Biot’s frequency
  ✓ Case study: Dynamic permeability

• Final remarks
Motivation

Partial saturated formations are of interest in a wide variety of scientific scenarios. Seismic methods have a great potential to help in the remote characterization of such environments.
Seismic methods for fluid detection

Elastodynamics:

Hooke’s law:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl},$$

Equilibrium equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{d^2 u_i}{dt^2}.$$

$$V_p = \sqrt{\frac{K + \frac{4}{3} \mu}{\rho}}, \quad V_s = \sqrt{\frac{\mu}{\rho}}.$$
Fluid effects on seismic velocities

P-wave velocity as a function of saturation for a carbonate simple of 0.3 porosity for two measuring frequencies.

\[
V_s(S_w) = \frac{\mu(S_w)}{\sqrt{\rho_b(S_w)}},
\]

\[
V_p(S_w) = \sqrt{\frac{K(S_w) + 4/3 \mu(S_w)}{\rho_b(S_w)}},
\]

Moduli are saturation dependent:

\[
\sigma_{ij} = C_{ijkl}(S_w) \epsilon_{kl}
\]
Saturation- and frequency-dependent effects

Seismic velocities are frequency-dependent:

\[ V_s(\omega, S_w), \quad V_p(\omega, S_w) \]

Implying that medium is effectively viscoelastic. Moduli are therefore complex-valued and frequency dependent, and Hooke’s law holds in the space-frequency domain:

\[ \hat{\sigma}_{ij}(\omega) = \hat{C}_{ijkl}(\omega, S_w) \hat{\varepsilon}_{kl}(\omega) \]

P-wave velocity as a function of saturation for a carbonate simple of 0.3 porosity for two measuring frequencies.
Fluid coupling effects on seismic signatures

What is causing this effect? There are mechanisms operating at different scales…
Fluid coupling effects on seismic signatures

What is causing this effect? There are mechanisms operating at different scales.
The critical frequency is given by:

\[ f_c \approx \frac{D}{2\pi l_{meso}^2} \]

where \( D \) is the diffusivity of the medium, and \( l_{meso} \) is the characteristic length of the heterogeneities.

Wave-induced fluid flow

Modified from Müller et al. (2010).
Modelling wave-induced fluid flow effects

We solve Biot’s (1941) poroelasticity equations in a representative elementary volume of the rock sample of interest under oscillatory forcing:

Illustration of (a) vertical, and (b) shear oscillatory relaxation tests to obtain the equivalent frequency-dependent moduli of the explored medium.

Plane-wave modulus

\[ M_c(\omega) = \frac{\langle \hat{\sigma}_{yy}(\omega) \rangle}{\langle \hat{\varepsilon}_{yy}(\omega) \rangle} \]

\[ Q_p^{-1}(\omega) = \frac{\Im\{M_c(\omega)\}}{\Re\{M_c(\omega)\}} \]

\[ V_p(\omega) = \left[ \Re\left( \frac{\langle \rho_b \rangle}{\sqrt{M_c(\omega)}} \right) \right]^{-1} \]
Saturation hysteresis and seismic signatures

P wave velocity vs. water saturation in a Berea sandstone (Knight & Nolen-Hoeksema, 1992)

Extensional attenuation vs. water saturation in a partially saturated sandstone (Yin et al., 1992)
Saturation hysteresis and seismic signatures

(a) Porosity map and (b) distribution of CO₂ and water during drainage and imbibition (Zhang et al., 2015).
Saturation hysteresis effects on the seismic signatures

Constitutive model of constrictive capillary tubes with a fractal pore size distribution (Soldi et al., 2017).
Saturation hysteresis effects on the seismic signatures

\[ \phi_{ij} \]

\[ \kappa_{ij} \]

\[ S_{wij}^i(p_c), S_{wij}^d(p_c) \]

Bourbié and Zinszner (1985)
Saturation hysteresis effects on the seismic signatures

\[ \phi_{ij}, \quad \kappa_{ij}, \quad S_{wij}^i(p_c), S_{wij}^d(p_c) \]

\[ x \text{ [m]}, \quad y \text{ [m]} \]

\[ \kappa \text{ [mD]} \]

\[ N \text{ of Cells} \]

\[ \phi, \quad \log(\kappa \text{ [mD]}) \]
Saturation hysteresis effects on the seismic signatures

\[ \phi_{ij} \]
\[ \kappa_{ij} \]
\[ S_{wij}^i(p_c), S_{wij}^d(p_c) \]
Saturation hysteresis effects on the seismic signatures

Taken from Solazzi et al (2019)
Saturation hysteresis effects on the seismic signatures

Taken from Solazzi et al (2019)
Fluid coupling effects on seismic waves

What is behind this process? There are mechanisms operating at different scales.

Taken from Muller et al (2010)
Squirt flow effects

Squirt flow mechanism:

- Microcracks can be present in grains.
- Fluid pressure diffusion occurs between these cracks and the connected pore space.

For most sedimentary water-saturated rocks this process occurs at frequencies larger $10^3$ Hz.

$$f_c \approx \frac{1}{2} \frac{K_S}{\eta} \left(\frac{h}{L}\right)^3$$
Squirt flow effects on partially saturated cracks

Fig 1: (a) Granite sample and (b) 3D CT-scan showing cracks in green (Fan et al., 2018). (c) Conceptual model.

Fig 2: (a) Sketch of the cracked rock and (b) blowup of a partially saturated crack. (c) Vertical profile, illustrating the distribution of the fluid phases within the crack.

Taken from Solazzi et al (2021)
Cracks behave as if saturated with a frequency-dependent effective fluid with a bulk modulus

\[ K_f^*(\omega) = S_w K_w \mathcal{T}_w(\omega) + (1 - S_w) K_n \mathcal{T}_n(\omega) \]

\( \mathcal{T}_n(\omega) \) and \( \mathcal{T}_w(\omega) \) are combinations of Bessel functions that include the crack geometrical properties, fluid compressibilities \( K_w \) and \( K_n \), and saturation \( S_w \).
Modulus dispersion and attenuation as functions of saturation

Fig 5: (a) Plane wave modulus $\Re\{c_{33}\}$ and (b) inverse quality factor $Q_p^{-1}$ as functions of saturation for vertically travelling P-waves.
Fluid coupling effects on seismic signatures

What is behind this process? There are mechanisms operating at different scales.
Biot’s intrinsic mechanism:

- Acceleration exerted by the passing wave.
- Flow is in the viscosity-dominated regime (Poiseuille flow) for \( f << f_B \).
- For much higher frequencies, the flow is inertia-dominated.

For most sedimentary water-saturated rocks this **normally process occurs at frequencies larger than** \( 10^4 \) Hz. Dominated by permeability and fluid viscosity.
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For most sedimentary water-saturated rocks this **normally** process occurs at frequencies larger than \( 10^4 \) Hz. Dominated by permeability and fluid viscosity.
Biot’s intrinsic mechanism: Dynamic permeability

The classic formulation of Darcy’s law only holds for viscous dominated flow:

$$\mathbf{w} = -\frac{\kappa_0}{\eta} \nabla p_f$$

If we want to use it for inertial flow, we need a frequency-dependent and complex-valued permeability:

$$\hat{\mathbf{w}}(\omega) = -\frac{\hat{\kappa}(\omega)}{\eta} \nabla \hat{p}_f(\omega)$$

Data from Smeulders et al (1992)
On the dynamic permeability of partially saturated porous media

While the frequency-dependence of permeability under fully saturated conditions has been studied for decades, the corresponding characteristics of partially saturated porous media remain unexplored.

We conceptualize the pore space as a bundle of tubes with a known pore size distribution.

We introduce a capillary pressure-saturation relationship to saturate the medium.

Frequency and saturation dependent permeability estimates

Taken from Solazzi et al. (2020)
On the dynamic permeability of partially saturated porous media

Effective permeability functions, as sensed by seismic waves in oscillatory flow, are functions of the saturation and of the frequency

Taken from Solazzi et al (2020)
Conclusions

• There are several mechanisms that induce frequency dependence in the effective elastic moduli of fluid saturated porous media.

• Wave-induced fluid flow in the mesoscopic scale is a fluid pressure diffusion process. It is modelled using Biot’s poroelasticity equations. Elastic moduli not only depend on the saturation state but, also, on the geometry of the fluid distributions.

• Squirt flow effects occur due to fluid pressure diffusion at the microscopic scale. It is modelled solving Navier-Stokes equations coupled with the elasticity equations. Elastic moduli depend on the geometry of the pores, fluid characteristic, and saturation state.

• Biot’s intrinsic mechanism is associated with fluid drag produced by an accelerated pore matrix. The effective permeabilities sensed by the wave in partially saturated conditions are saturation and frequency dependent.

• Understanding the physical reasons behind these processes may permit a better characterization of the hydraulic and mechanical properties of the explored media using seismic methods.
Thank you very much!
Fluid coupling effects on seismic signatures

So... what happens to the elastic moduli? It appears they are also frequency dependent...
Seismic Method

Elastodynamics:

Hooke’s law:
\[ \sigma_{ij} = C_{ijkl} \varepsilon_{kl} \]

Equilibrium equation:
\[ \frac{\partial \sigma_{ij}}{\partial x_j} = \rho \frac{d^2 u_i}{dt^2} \]

\[ V_p = \sqrt{\frac{K + \frac{4}{3} \mu}{\rho}}, \quad V_s = \sqrt{\frac{\mu}{\rho}}. \]

Gorney et al. (2007).
Biot’s theory of poroelasticity

Variables of Biot’s theory:

\( u \): Solid displacement.

\( w \): Relative fluid displacement

\( \zeta = -\nabla \cdot w \): Variation in the fluid content

\( p_f \): Fluid pressure

\( \sigma \): Total stress tensor

\( \varepsilon \): Strain tensor

Giorgiadis et al. (2013)
Biot’s theory of poroelasticity

Constitutive relations

\[ \sigma_{ij} = 2\mu \epsilon_{ij} + \delta_{ij} (\lambda_c \epsilon_{kk} - \alpha M \zeta), \]
\[ p_f = -\alpha M \epsilon_{kk} + M \zeta. \]

Biot’s poroelastic equations:

\[ \nabla \cdot \sigma = \rho_b \ddot{u} + \rho_f \ddot{w}, \]
\[ -\nabla p_f = \rho_f \ddot{u} + g \ddot{w} + \frac{\eta}{\kappa} \dddot{w}. \]

Coefficients

\[ \begin{align*}
\alpha & \quad \mu \\
M & \quad \lambda_c
\end{align*} \]

Supports 3 types of waves

\[ \begin{align*}
P & \quad P_2 \\
S
\end{align*} \]
Biot’s theory of poroelasticity

Constitutive relations

\[ \sigma_{ij} = 2\mu \varepsilon_{ij} + \delta_{ij}(\lambda_c \varepsilon_{kk} - \alpha M \zeta), \]
\[ p_f = -\alpha M \varepsilon_{kk} + M \zeta. \]

Coefficients

\[ \begin{align*}
\alpha & \quad \mu \\
M & \quad \lambda_c
\end{align*} \]

Biot’s quasi-static equations:

\[ \nabla \cdot \sigma = 0, \]
\[ -\nabla p_f = \frac{\eta}{\kappa} \dot{\omega}. \]

Supports 3 types of waves

P, P_2, S

Equilibrium of forces and Darcy equation
Validation of the analytical solution

Fig 3: (a) Inverse quality factor $Q_p^{-1}$ and (b) plane wave modulus $\Re\{c_{33}\}$ as functions of frequency for vertically travelling P-waves. Background properties are taken from a Westerly Granite characterized by a fracture density of $\varepsilon = 4.6 \times 10^{-3}$. Cracks have an aspect ratio $\alpha = 3.6 \times 10^{-3}$ and an aperture $h_0 = 10 \mu m$. Fluids properties are those of glycerin and air.
Fluid coupling effects on seismic signatures

Do we also observe this in the lab?

Forced oscillation apparatus, SINTEF, Norway (Lozovyi et al, 2019)

Frequency ranges of different techniques to measure attenuation and dispersion of (Subramaniyan et al, 2014).
Gassmann’s equations

Gassmann’s (1951) model:

\[ K_{sat} = K_{dry} + \frac{(1 - K_{dry}/K_g)^2}{\phi K_f + (1 - \phi) + \frac{K_{dry}^2}{K_g}}, \]

\[ \mu_{sat} = \mu_{dry}, \]

\[ V_s(f) = \sqrt{\frac{\mu}{\rho_b}}, \]

\[ V_p(f) = \sqrt{\frac{K + 4/3\mu}{\rho_b}}, \]

Gassmann’s (1951) low-frequency model
Fluid effects on seismic signatures

There are three main dissipation processes associated with fluid/solid coupling:

- **Intrinsic Biot’s mechanism**
  - Take place in a homogeneous and fully-saturated porous media, and are modified in presence of partial saturation.

- **Squirt flow mechanism**

- **Fluid pressure diffusion in the mesoscopic scale**
  - Does not take place in homogeneous and fully-saturated media. However, it arises in partial saturation scenarios!
Fluid content and Seismic Signatures

Partially saturated medium

Water
Air
Solid Grains

Mechanical properties

Seismic Signatures

Hydraulic properties
Fluid coupling effects on seismic signatures

So… what happens to the elastic moduli?

OMG, they are also frequency dependent!

Illustrative example of experimentally retrieved frequency-dependent Young modulus $E$ and inverse quality factor $1/Q$ in a Berea sandstone with a saturation of 99% (Chapman et al, 2016).