Numerical modeling of cracking process in partially saturated porous media and application to rainfall-induced slope instability analysis

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Outline

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PART ONE

BACKGROUND

Introduction

Landslide

Predisposing factors	• Lithology • Faults • Land use	Easily assessed by spatial analysis techniques *(Ayalew et al. 2005)*
Triggering factors	• Rainfall • Snowmelt • Earthquakes	Difficult to estimate at a regional scale *(Griffiths 2014)*

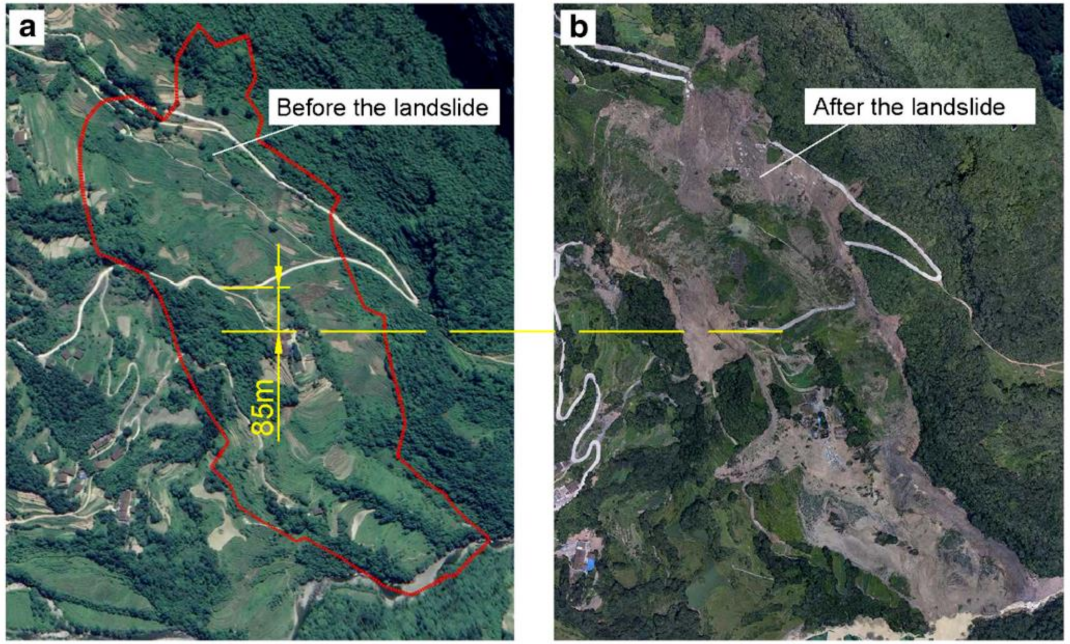

Figure –Zhongbao lanslide at Wulong, China
(CHEN et al. 2021)

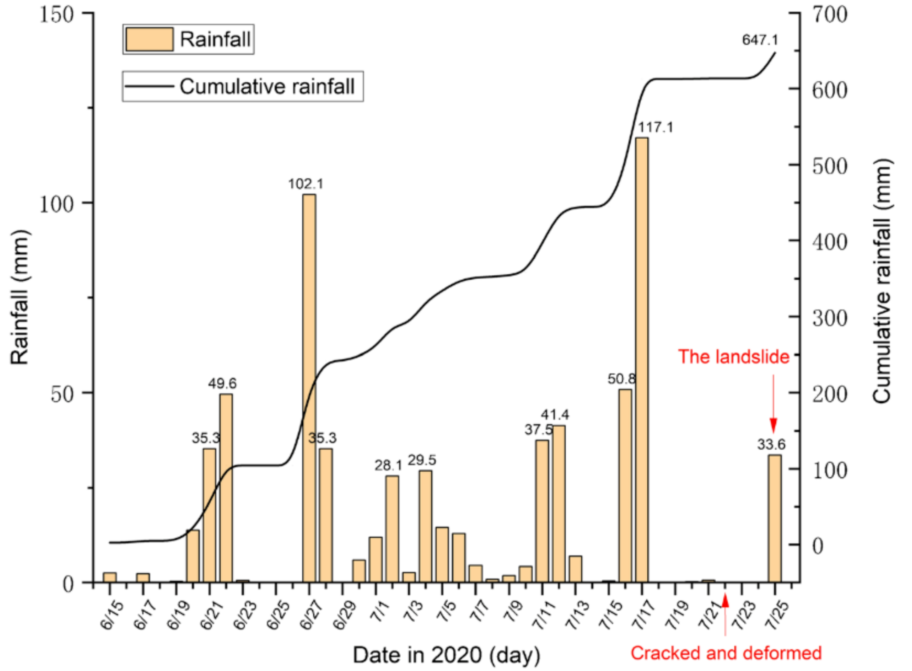

Figure –Rainfall monitoring data
(CHEN et al. 2021)

Objectives

Triggering factors: **Rainfall** → Material weakening / Induced deformation → Crack evolution

Approaches for slope stability analysis:

- Limit equilibrium method (LEM) → Safety factor
- Numerical modelling, progressive deformation and instability e.g. Finite element method (FEM) → Safety factor

Methodology: Phase-field Method (PFM)

Regularized crack topology:

$$A_\Gamma = \int_\Gamma \mathrm{d}A \cong \int_\Omega \gamma_d(d, \nabla d)\, \mathrm{d}V$$

- Predict not only **crack initiation** but also the **crack propagation path**;
- Deal with merging and branching of **multiple cracks**;
- Easy to incorporate the **multi-field physics**

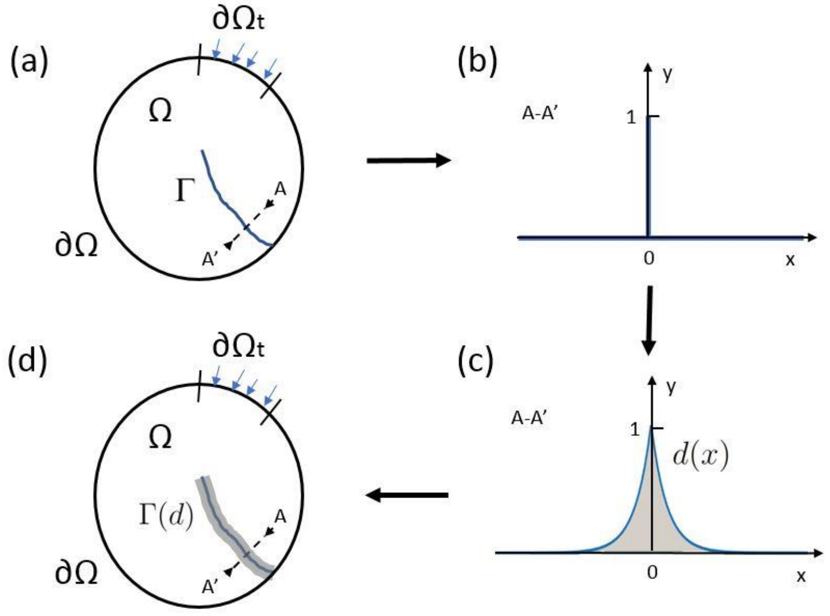

Figure –Regularized crack topology

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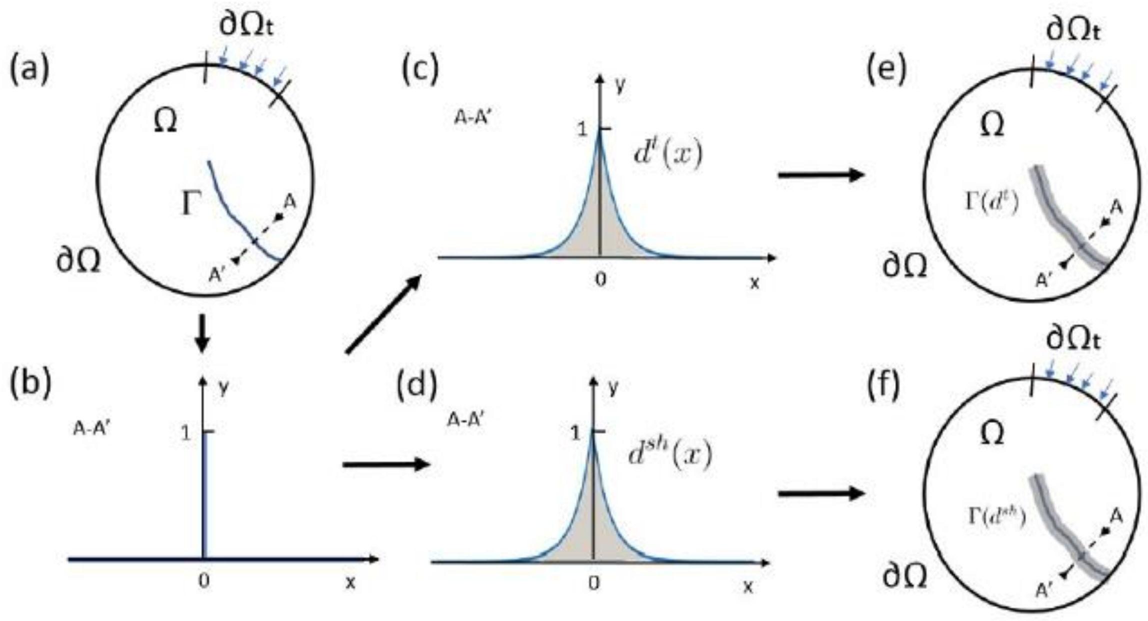
PART TWO

PHASE-FIELD FORMULATIONS FOR PARTIALLY SATURATED MEDIA

Regularized crack fields

Two independent variables d^t and d^s to approximate the crack surface area:



The total crack surface density : $\gamma_d(d, \nabla d) = \gamma_d^t(d^t, \nabla d^t) + \gamma_d^s(d^s, \nabla d^s)$

Tensile crack density $\dfrac{(d^t)^2}{2l_d} + \dfrac{l_d}{2} \nabla d^t . \nabla d^t$

$\dfrac{(d^s)^2}{2l_d} + \dfrac{l_d}{2} \nabla d^s . \nabla d^s$ Compressive-shear crack density

The energy dissipated by cracks :

$$D(d^s, d^t) = \int_\Omega \left[g_c^t \gamma_d^t(d^t, \nabla d^t) + g_c^s \gamma_d^s(d^s, \nabla d^s) \right] dV$$

Constitutive relations of undamaged porous media

Poroelastic model for the undamaged material (Coussy, 2010) :

$$d\boldsymbol{\sigma}^0 = d\boldsymbol{\sigma}^{b0} - bS_w dp_w I$$

$$dp_w = \mathrm{M}_{ww}\left[-bS_w d\boldsymbol{\varepsilon}_v + \left(\frac{dm_w}{\rho_w}\right)\right]$$

- The capillary pressure ($p_g = p_{atm} = 0$):

$$p_c = -p_w$$

- The extended Bishop's effective stress (Bishop, 1959) :

$$d\boldsymbol{\sigma}^{b0} = d\boldsymbol{\sigma}^0 + bS_w dp_w I = \mathbb{C}^{b0} : d\boldsymbol{\varepsilon}$$

- The water saturation degree (van Genuchten, 1980) :

$$S_w = S_r + S_e(1 - S_r)$$

$$S_e = \left[1 + \left(\frac{p_c}{p_{cr}}\right)^n\right]^{-m}$$

The total energy functional of partially saturated cracked material

$$E(\boldsymbol{\varepsilon}, m_w, d^t, d^s) = \int_\Omega \psi(\boldsymbol{\varepsilon}, m_w, d^t, d^s)\mathrm{d}V + \int_\Omega \mathcal{D}(d^t, d^s)\,\mathrm{d}V$$

stored energy — cracking dissipation

$$\psi(\boldsymbol{\varepsilon}, m_w, d^t, d^s) = \psi_{eff}(\boldsymbol{\varepsilon}, d^t, d^s) + \psi_{fluids}(\boldsymbol{\varepsilon}, m_w)$$

Skeleton deformation — Fluid mass change

Stored energy for partially saturated media with cracks

- The stored elastic energy of porous medium:

$$\psi_{eff}(\boldsymbol{\varepsilon}, d^t, d^s) = g(d^t)\,W_+^b(\boldsymbol{\varepsilon}) + g(d^s)\,W_-^b(\boldsymbol{\varepsilon})$$

$$W_+^b(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\sigma}_+^{\mathrm{b}} : \boldsymbol{\varepsilon}$$

Tensile crack driving energy

$$W_-^b(\boldsymbol{\varepsilon}) = \frac{1}{2}\boldsymbol{\sigma}_-^{\mathrm{b}} : \boldsymbol{\varepsilon}$$

Shear crack driving energy

- The degradation function (Miehe et al. 2010) :

$$g(d^\alpha) = (1 - d^\alpha)^2$$

- The Decomposition of effective stress tensors :

$$\boldsymbol{\sigma}_\pm^b = \sum_{a=1}^{3} \langle \sigma_a \rangle_\pm \boldsymbol{n}_a \otimes \boldsymbol{n}_a$$

- The energy due to fluid mass change :

$$\psi_{fluids}(\boldsymbol{\varepsilon}, m_w, d^t, d^s) \equiv \psi_{fluids}(\boldsymbol{\varepsilon}, m_w) = \frac{1}{2} M_{ww} \left[b S_w \boldsymbol{\varepsilon}_{\mathrm{v}} - \left(\frac{m}{\rho}\right)_w \right]^2$$

$$\Pi\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) = \dot{E}\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) - \dot{P}_{ext} = 0$$

Governing equations for phase-field variables

$$-g_t'(d^t) W_+^b - g_c^t \left[\frac{d^t}{l} - l\,\mathrm{div}(\nabla d^t)\right] = 0$$

$$W_+^e = \frac{1}{2}\boldsymbol{\sigma}_+ : \boldsymbol{\varepsilon}^e$$

$$\mathcal{H}^t = \max_{t\in[0,t]} W_+^e$$

$$-2(1-d^t)\mathcal{H}^t - g_c^t \left[\frac{d^t}{l} - l\,div(\nabla d^t)\right] = 0$$

$$-g_s'(d^s) W_-^b - g_c^s \left[\frac{d^s}{l} - l\,\mathrm{div}(\nabla d^s)\right] = 0$$

$$W_-^s = \frac{1}{2G}\left\langle \frac{\sigma_3^- - \sigma_1^-}{2\cos\varphi} + \frac{\sigma_3^- + \sigma_1^-}{2}\tan\varphi - c \right\rangle_+^2$$

$$\mathcal{H}^s = \max_{t\in[0,t]} W_-^s$$

$$-2(1-d^s)\mathcal{H}^s - g_c^s \left[\frac{d^s}{l} - l\,div(\nabla d^s)\right] = 0$$

$$\Pi\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) = \dot{E}\left(\dot{\boldsymbol{u}}, \dot{m}_w, \dot{m}_g, \dot{d}^t, \dot{d}^s\right) - \dot{P}_{ext} = 0$$

Hydro-mechanics coupling functions for partially saturated medium

$$p_w - p_{w0} = M_w\left[-b_w \boldsymbol{\varepsilon}\,\mathrm{I} + \frac{m_w}{\rho_w}\right]$$

Darcy's law and mass conservation

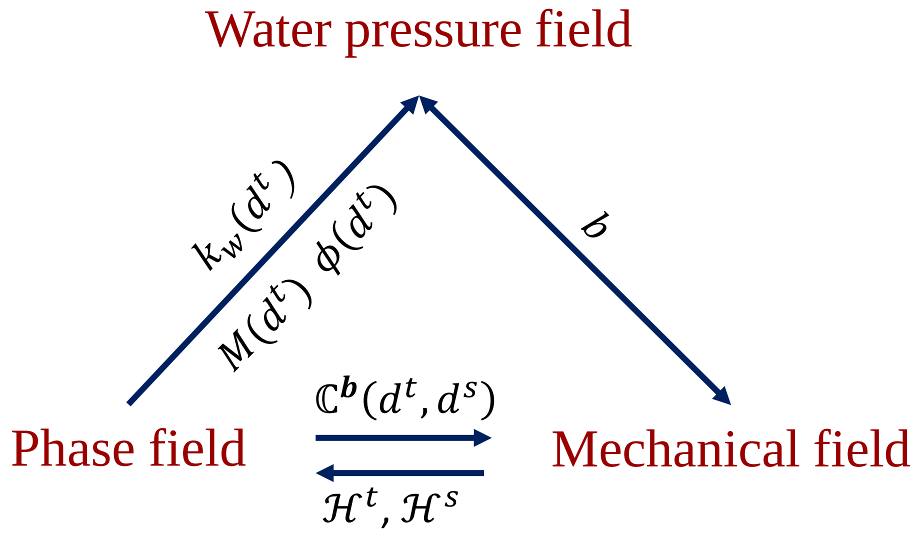
$$bS_w\dot{\varepsilon}_v + \frac{1}{M}\dot{p}_w = \frac{k_r k_w}{\mu_w}\cdot \mathrm{div}(\nabla p_w - \rho_w \vec{g})$$

$$\mathrm{div}(\boldsymbol{\sigma}) + \vec{f} = 0$$

$$\boldsymbol{\sigma} - \boldsymbol{\sigma}_0 = \mathbb{C}^b(d^t, d^s) : \boldsymbol{\varepsilon} - bS_w(p_w - p_{w0})\mathrm{I}$$

Effects of phase field on hydraulic parameters:

- Permeability: $k_w(d^t) = k_w^0 \exp(d^t)$
- Porosity: $\phi(d^t) = \phi^0 + (1 - \phi^0)\, d^t$
- Scalar parameter: $\frac{1}{M(d^t)} = \frac{S_l^2\,[b - \phi(d^t)]}{K_s} + \frac{S_l \phi(d^t)}{K_f} - \phi(d^t)\frac{\partial S_l}{\partial p_c}$



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PART THREE

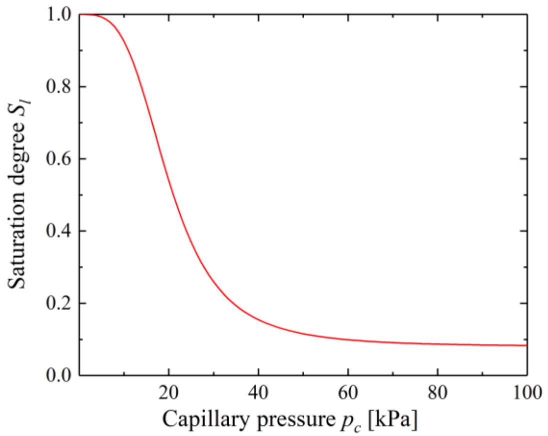
NUMERICAL MODELING

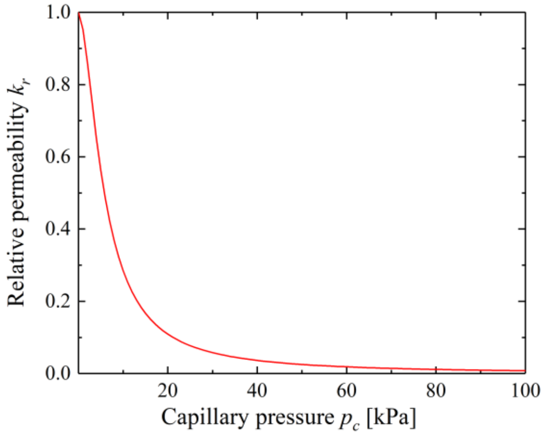
Analysis of rainfall induced landslides

Hydro-mechanical parameters:

λ	μ	K_w	ϕ	b	k_{pl}
2.9 GPa	*0.7 GPa*	2.2×10^9 *Pa*	*0.38*	*1.0*	5×10^{-12} m^2

Water retention and relative permeability curves





Phase-field parameters:

Critical energy g_c^t	Critical energy g_c^s	Crack length scale l
224 N/m	*364 N/m*	*0.25m*

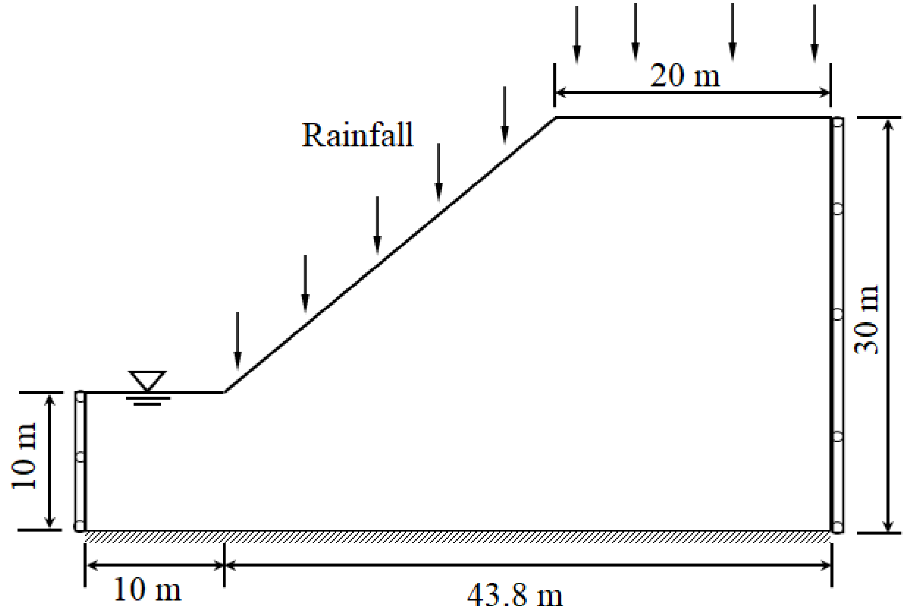


Figure - Boundary conditions

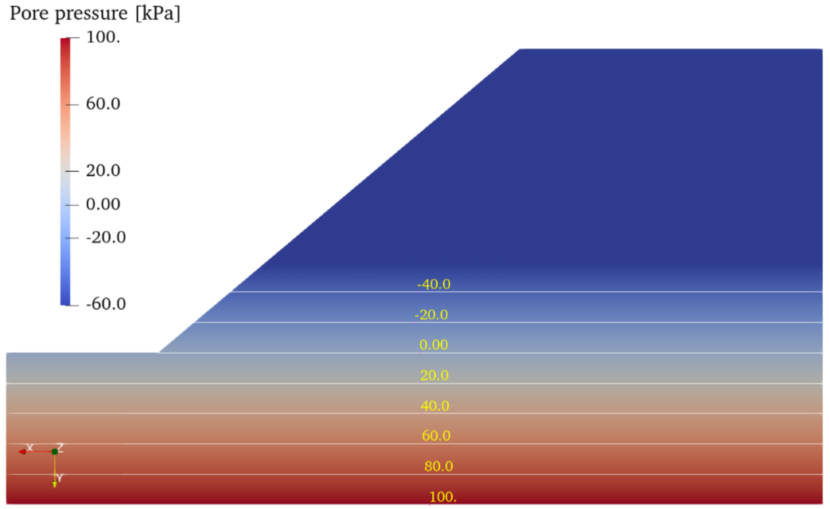


Figure - Initial distribution of pore pressure

Analysis of rainfall induced landslides

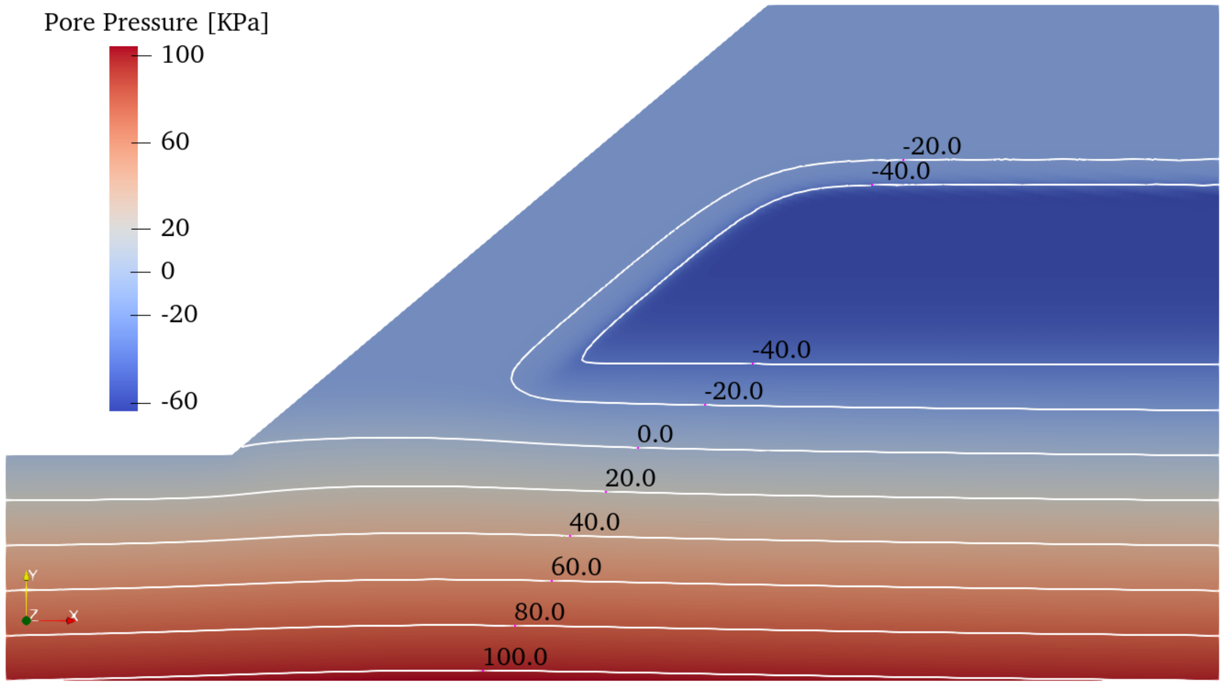


Figure - Distribution of pore pressure without damage (after 66h)

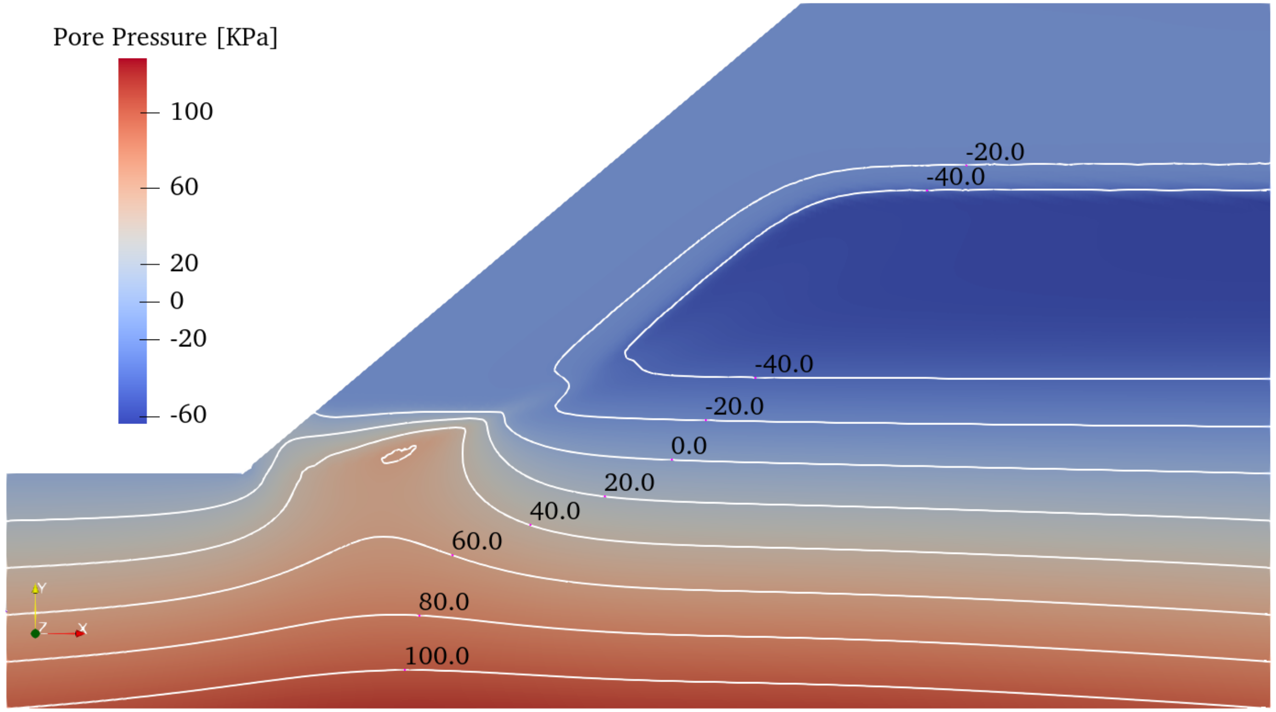


Figure - Distribution of pore pressure when slope failure occurs (after 65.5h)

Rainfall infiltration:

- Increment of underground water table
- Partially saturated ⟶ fully saturated (toe of the slope)

Pore pressure ⟷ cracks

Analysis of rainfall induced landslides

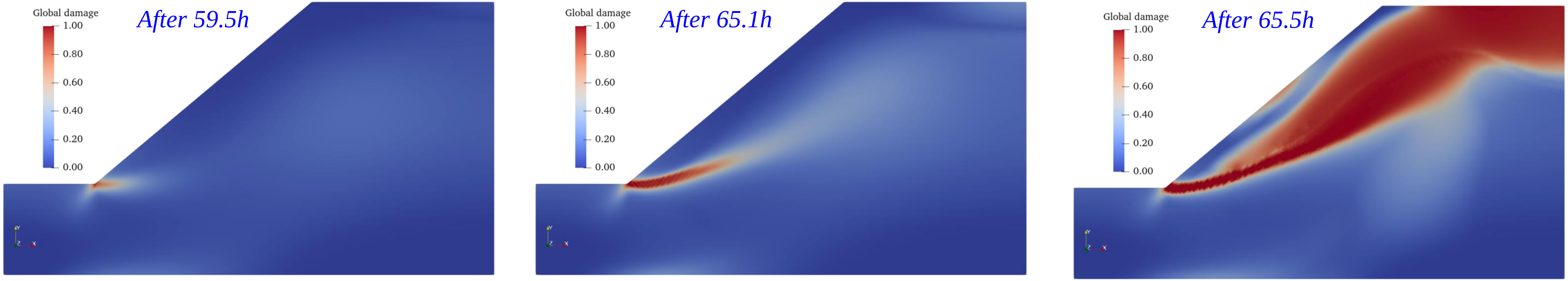


Figure - Distribution of global damage

- Onset of cracks: Around the toe of the slope
- Cracks path: Toe of slope ⟶ top of slope

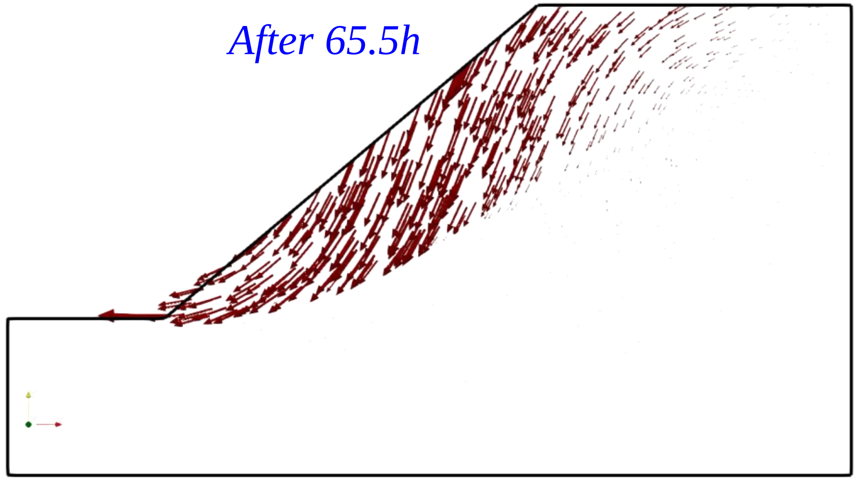
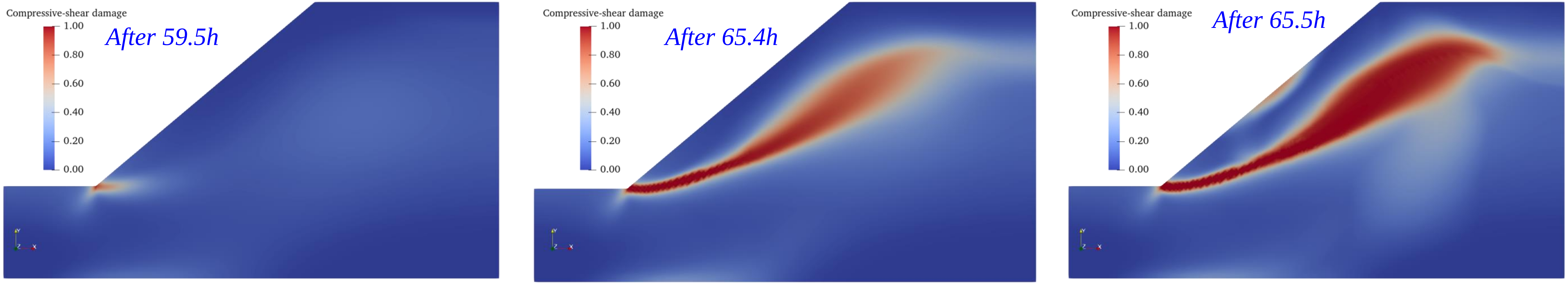
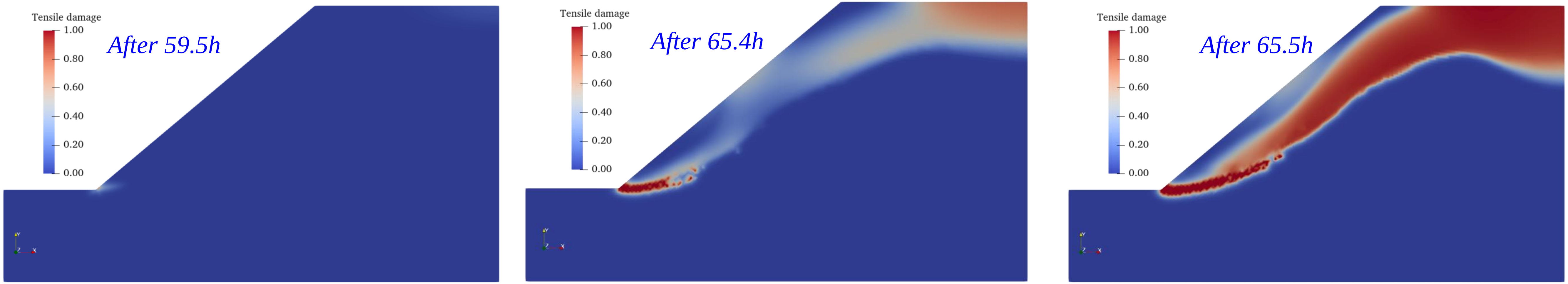


Figure - Displacement vector

Analysis of rainfall induced landslides

Compressive- shear cracks





Tensile cracks

Analysis of rainfall induced landslides

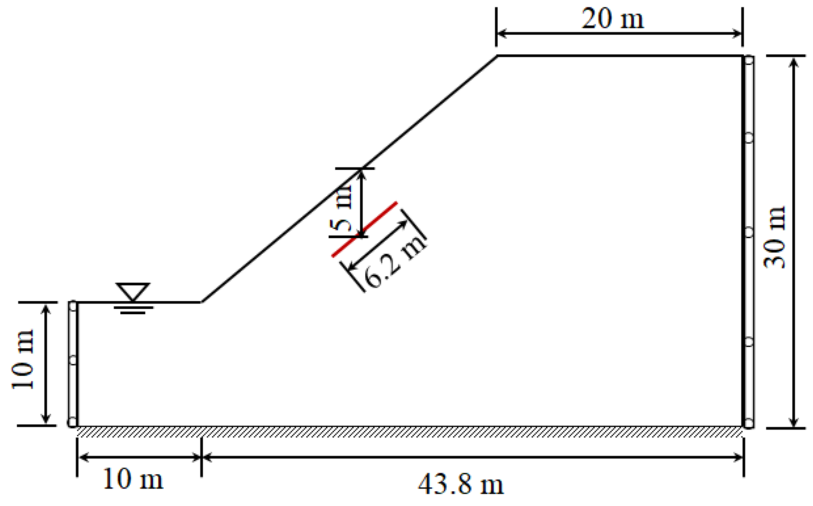


Figure – Distribution of pre-crack

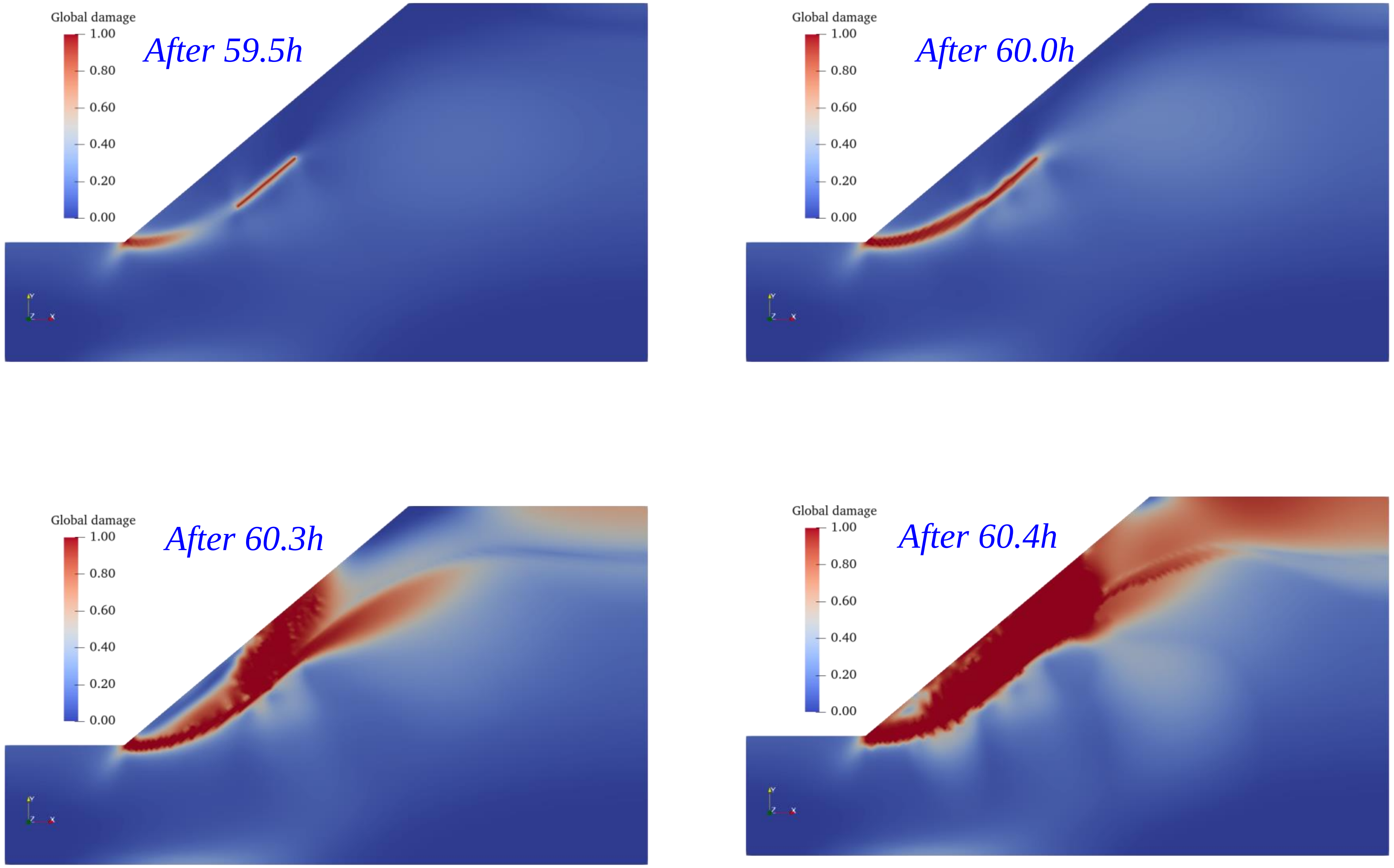


Figure - Distribution of global damage

Influences of pre-crack:

- Growth of cracks
- Two-step failure pattern

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PART FOUR

CONCLUSIONS AND PERSPECTIVES

Conclusions

- The proposed method is able to describe the initiation and propagation of localized damage zones and cracks due to rainfall.
- It was found that the shear cracking was the principal failure mechanism of landslides.
- The existence of initial weak zones and fractures enhances the failure process and also affects the cracking pattern

Perspectives

- Application the proposed numerical method into analysis of reality landslides;
- Considering the material in a slope as a heterogeneous material;
- Proposing a time-dependent phase-field method to simulate the long-term behavior of the slope;
- Taking into account hydrodynamic effects

Thank you for your attendance !

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