CFMR

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Réunion technique

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Post-closure hydromechanical behaviour of a backfilled cavity

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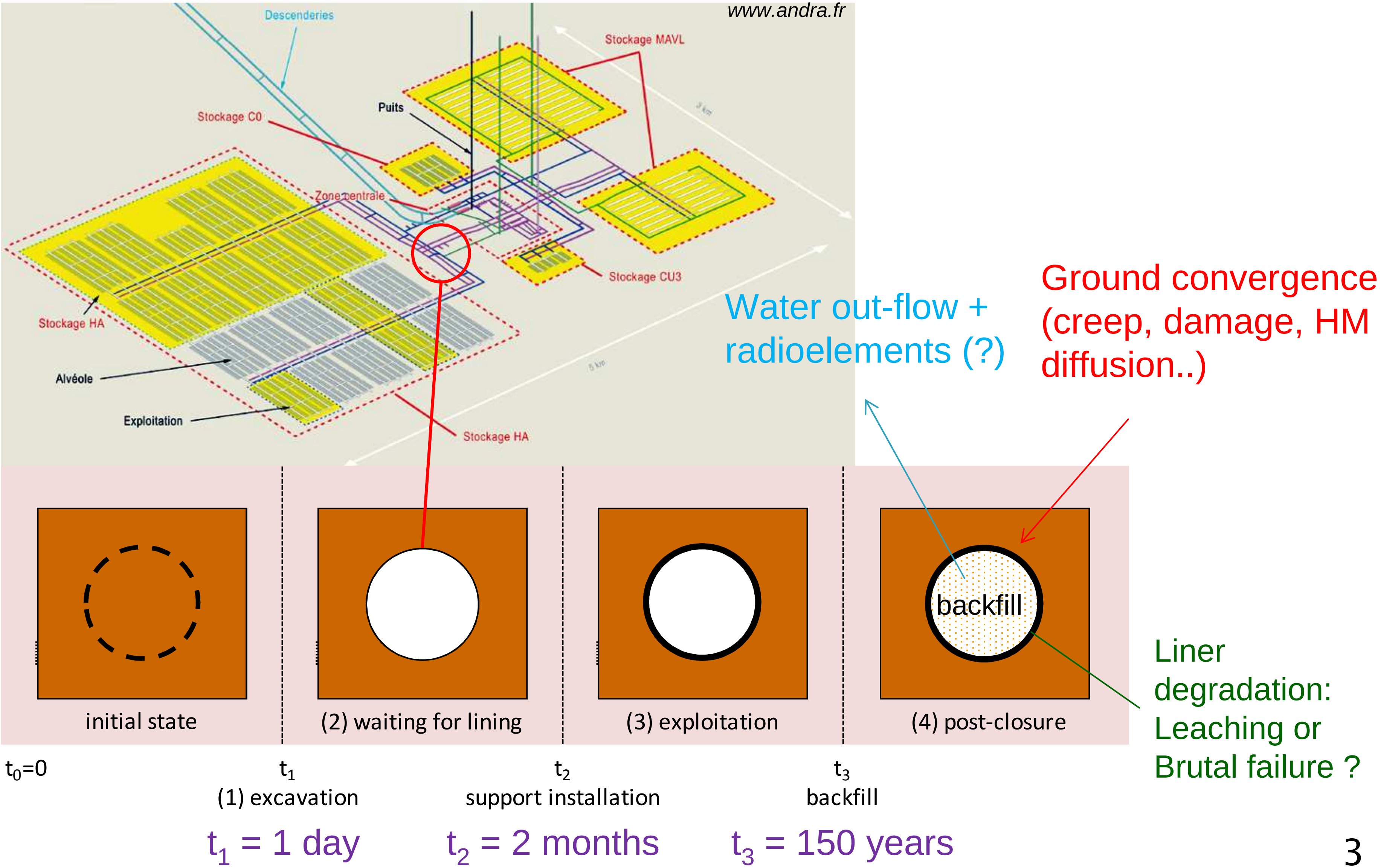
Plan

- Problem setting
- Linear behaviour – Analytical approach
  poroelasticity
  poroviscoelasticity
- Non linear behaviour – Numerical approach
  Poro-viscoplasticity + isotropic damage
  poro-viscoplasticity + anisotropic damage
- Conclusions

Idealized problem



Methodology

1. Linear material behaviour :

Poroelasticity

poro-viscoelasticity

Analytical ou Quasi-analytical solutions

(Benchmarks)

↓

2. Non-linear behaviors

Poro-viscoplasticity,

Iso/aniso damage, …

Numerical solutions

- Wong, H., Morvan, M., Deleruyelle, F. & Leo, C.J. 2008a Analytical study of mine closure behaviour in a poro-elastic medium, *Computers and Geotechnics* 35(5):645-654.
- Wong, H., Morvan, M., Deleruyelle, F. & Leo, C.J. 2008b Analytical study of mine closure behaviour in a poro-visco-elastic medium, *International Journal for Numerical and Analytical Methods in Geomechanics* 32(14):1737-1761.
- Dufour, N., Leo, C.J., Deleruyelle, F. & Wong, H. 2009 Hydromechanical responses of a decommissioned backfilled tunnel drilled into a poro-viscoelastic medium, *Soils and Foundations* 49(4):495-507.
- *Dufour, N., Wong, H., Deleruyelle, F. & Leo, C.J 2010 Hydromechanical post-closure behaviour of a deep tunnel taking into account a simplified life cycle, Int. J of Geomechanics, in press.*
- + …

1. Linear material behaviour :

Poroelasticity

Poro-viscoelasticity

Analytical or quasi-analytical solutions

Principal equations Poroelasticity

- Behaviour laws *(Coussy, 2004)*

$$\boldsymbol{\sigma} - \boldsymbol{\sigma}^0 = \left(K - \frac{2G}{3}\right)\epsilon(\boldsymbol{u})\mathbf{1} + 2G\boldsymbol{\varepsilon}(\boldsymbol{u}) - b(p - p_0)\mathbf{1}$$

$$\Phi - \Phi^0 = b\epsilon(\boldsymbol{u}) + \frac{p - p_0}{N}$$

- Darcy's law $\boldsymbol{V^D} = \lambda_h(-\boldsymbol{grad}(p) + \rho_f \boldsymbol{g})$

- Equilibrium $div(\boldsymbol{\sigma} - \boldsymbol{\sigma}^0) = 0$

2-fields problem (u, p) : cylindrical or spherical symmetry

$$\begin{cases} \left(K + \frac{4}{3}G\right)\epsilon(u) - (p - p_0) = 0 \\ \frac{\partial(p - p_0)}{\partial t} = k_h \Delta(p - p_0) \end{cases}$$

$$\epsilon(u) = \frac{\partial u}{\partial r} + \frac{u}{r}$$ Circular tunnels

$$\epsilon(u) = \frac{\partial u}{\partial r} + \frac{2u}{r}$$ Spherical cavities

Analytical solution : tools

Laplace Transform

Partial differential eqs

$$\begin{cases}\left(K+\frac{4}{3}G\right)\epsilon(u)-(p-p_0)=0\\ \frac{\partial(p-p_0)}{\partial t}=k_h\Delta(p-p_0)\end{cases}$$

$\mathscr{L}$ →

Ordinary differential eqs

$$\begin{cases}\frac{\partial^2\bar{p}(r,s)}{\partial r^2}+\frac{1}{r}\frac{\partial\bar{p}(r,s)}{\partial r}-q^2\bar{p}(r,s)=\frac{-q^2}{s}p_0(r)\\ \frac{\partial\bar{u}(r,s)}{\partial r}+\frac{\bar{u}(r,s)}{r}=\frac{1}{\omega}\left(\bar{p}(r,s)-\frac{p_0(r)}{s}\right)\end{cases}$$

↓

$$\begin{cases}\bar{u}(r,s)\\ \bar{p}(r,s)\end{cases}$$

← $\mathscr{L}^{-1}$

$$\begin{cases}u(r,t)\\ p(r,t)\end{cases}$$

Temporal domain

Laplace's transforms domain

Stehfest's (1970)

$$f(t)\cong\frac{\ln 2}{t}\sum_{n=1}^{N}C_n\bar{f}\left(n\frac{\ln 2}{t}\right)$$

$$C_n=(-1)^{n+\frac{N}{2}}\sum_{k=Int\left(\frac{n+1}{2}\right)}^{\min\left(n,\frac{N}{2}\right)}\frac{k^{\frac{N}{2}}(2k)!}{\left(\frac{N}{2}-k\right)!\,k!\,(k-1)!\,(n-k)!\,(2k-n)!}$$

Analytical solution
Poroelastic rock, spherical cavities

$$u'(r',t') = \frac{(p_0'-1)}{\omega r'^2}\Big[g(r',t',\Omega_1) - g(r',t',\Omega_2) - A\varphi(r',t',\Omega_1) - B\varphi(r',t',\Omega_2) - C\varphi\left(r',t',-i\sqrt{\kappa'}\right) - D\varphi\left(r',t',i\sqrt{\kappa'}\right)\Big]$$

$$p'(r',t') - p_0' = \frac{1-p_0'}{r'}\kappa'\left\{A^*\varphi(r',t',\Omega_1) + B^*\varphi(r',t',\Omega_2) + C^*\varphi\left(r',t',-i\sqrt{\kappa'}\right) + D^*\varphi\left(r',t',i\sqrt{\kappa'}\right)\right\}$$

with
$$g(r',t',h) = \frac{1}{\Omega_1-\Omega_2}\left\{\left(\frac{1}{h}+r'\right)\exp\left(-h(r'-1)+h^2t'\right)erfc\left(\frac{r'-1}{2\sqrt{t'}}-h\sqrt{t'}\right) - \frac{1}{h}erfc\left(\frac{r'-1}{2\sqrt{t'}}\right)\right\}$$

$$\varphi(r',t',h) = L^{-1}\left\{\frac{e^{-q(r'-1)}}{(q-h)}\right\}$$

$$= \frac{1}{\sqrt{\pi t'}}\exp\left(-\frac{(r'-1)^2}{4t'}\right) + h\exp\left(-h(r'-1)+h^2t^2\right)erfc\left(\frac{r'-1}{2\sqrt{t'}}-h\sqrt{t'}\right)$$

Wong, H., Morvan, M., Deleruyelle, F. & Leo, C.J. 2008a Analytical study of mine closure behaviour in a poro-elastic medium, Computers and Geotechnics 35(5):645-654.

Quasi-analytical solution phases 1, 2, 3

- **1st stage : instantaneous excavation**

$$p^{(1)}(r) = p_0 \qquad\qquad u^{(1)}(r) = -\frac{\Sigma_0}{2G}\,\frac{a^2}{r}$$

- **2nd et 3rd stages : unlined gallery, pore pressure dissipation**

$$p^{(2)}(r,s) = \frac{p_0}{s}\left[1 - \frac{K_0(qr)}{K_0(qa)}\right]$$

$$\overline{u^{(2)}}(r,s) = \frac{p_0}{qs\omega K_0(qa)}\left[K_1(qr) - \frac{aK_1(qa)}{r}\right] - \frac{a^2}{r}\,\frac{\Sigma_0}{2Gs}$$

Quasi-analytical solution
Stage 4: backfill + post-closure

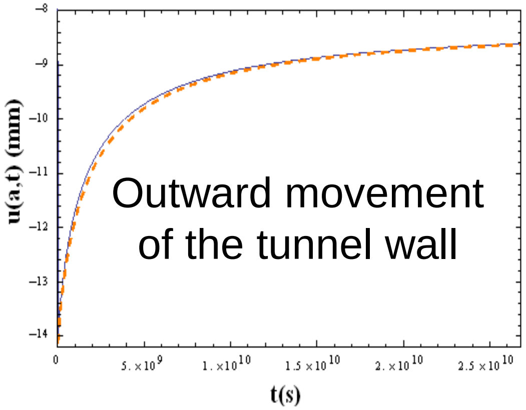
Numerical inversion (Stehfest, 1970) → $p^{(4)}(r,t)$, $u^{(4)}(r,t)$

$$\overline{\hat{p}^{(4)}}(r,s) \approx A^{(4)}(s)K_0(qr) + p_0 \sum_{n=1}^{N} \frac{C_n}{n}\left[\frac{1}{s} - \frac{t_3}{(st_3 - n\ln 2)}\frac{K_0(\alpha_n r)}{K_0(\alpha_n a)}\right]$$

$$\overline{\hat{u}^{(4)}}(r,s) \approx \frac{C^{(4)}(s)}{r} + \frac{r}{2\omega}\left(A^{(4)}(s)K_0(qr) - p_0 t_3 \sum_{n=1}^{N} \frac{C_n}{n(st_3 - n\ln 2)}\frac{K_0(\alpha_n r)}{K_0(\alpha_n a)}\right)$$

$$A^{(4)}(s) = \frac{p_0}{a\omega s K_0(qa) + 2G\sqrt{k_h s}K_1(qa)}\left[\sum_{n=1}^{N} \frac{C_n}{(st_3 - n\ln 2)}\left(\omega a \ln 2 + 2G\sqrt{\frac{k_h \ln 2 t_3}{n}}\frac{K_1(\alpha_n a)}{K_0(\alpha_n a)}\right)\right]$$

$$C^{(4)}(s) \approx \frac{a^2}{2}\left[\left(\frac{1}{\omega} - \frac{1}{G}\right)\left(p_0 t_3 \sum_{n=1}^{N} \frac{C_n}{n(st_3 - n\ln 2)} - A^{(4)}(s)K_0(qa)\right) + \frac{1}{Gs}(-\bar{\Sigma}_0 + p_0)\right]$$



Dufour, N., Wong, H., Deleruyelle, F. & Leo, C.J 2010 Hydromechanical post-closure behaviour of a deep tunnel taking into account a simplified life cycle, Int. J of Geomaterials, in press.

Analytical solution
Poroviscoelasticity, spherical cavities

Stress-strain:

$$(\sigma - \sigma_0) + (p - p_0) = K \hat{\otimes} \in$$

$$s_{ij} - s^0_{ij} = 2\mu \hat{\otimes} e_{ij}$$

Stieltjes convolution :

$$x \hat{\otimes} y \equiv \int_{0^-}^{t} x(t-\tau)\dot{y}(\tau)d\tau$$

Elastic « functions »:

$$K(t) = \left[K_\infty - (K_\infty - K_0)\exp\left(-\frac{t}{\tau_r}\right)\right] H(t)$$

$$\mu(t) = \left[\mu_\infty - (\mu_\infty - \mu_0)\exp\left(-\frac{t}{\theta_r}\right)\right] H(t)$$

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Analytical solution
Poroviscoelasticity, spherical cavities

$$\frac{r'}{\kappa'\omega'_e(1-p'_0)} p'-p'_0(r',t') = H_1(r',t') \otimes \sum_{k=1}^{6} A_k e^{-y_k t'} + H_2(r',t') \otimes \sum_{k=1}^{6} B_k e^{-y_k t'}$$

$$H_1(t') = L^{-1}\left[e^{-q'(r'-1)}\right] = e^{-v'_c t'}\left\{ f(t') - \int_0^{t'} \sqrt{\frac{v'_c(v'_r - v'_c)\tau}{t'-\tau}}\, J_1\left(2\sqrt{v'_c(v'_r - v'_c)\tau(t'-\tau)}\right) f(\tau)\, d\tau \right\}$$

$$\textit{with} \qquad f(t') = \frac{r'-1}{2\sqrt{\pi t'^3}} \exp\left[(2v'_c - v'_r)t' - \frac{(r'-1)^2}{4t'}\right]$$

$$H_2(t') = L^{-1}\left[q' e^{-q'(r'-1)}\right] = e^{-v'_c t'}\left\{ g(t') - \int_0^{t'} \sqrt{\frac{v'_c(v'_r - v'_c)\tau}{t'-\tau}}\, J_1\left(2\sqrt{v'_c(v'_r - v'_c)\tau(t'-\tau)}\right) f(\tau)\, d\tau \right\}$$

$$\textit{with} \qquad g(t') = \frac{1}{4\sqrt{\pi}} \frac{\left[(r'-1)^2 - 2t'\right]}{t^{5/2}} \exp\left[(2v'_c - v'_r)t' - \frac{(r'-1)^2}{4t'}\right]$$

Wong, H., Morvan, M., Deleruyelle, F. & Leo, C.J. 2008a Analytical study of mine closure behaviour in a poro-visco-elastic medium, International Journal for Numerical and Analytical Methods in Geomechanics 32(14):1737-1761.

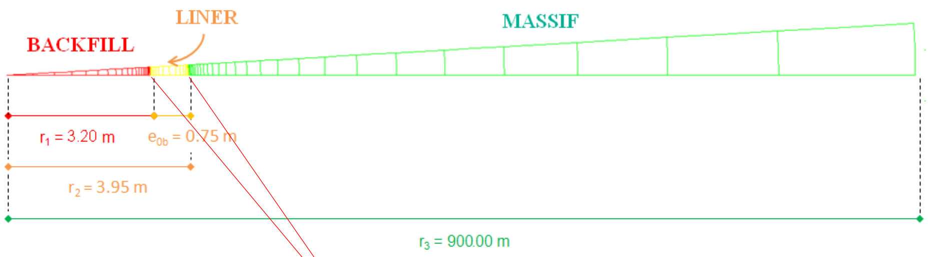
2. Non-linear behaviors

Poro-viscoplasticity,

Iso/anisotropic damage, …

Numerical solutions

Cast3M : superposition of models



1 FEM mesh
1 displacement field

CAST3M
Huge library of
material models

(1) Monophasic model

(2) Poroelastic model

$$d\sigma_{ij} = D^{T}_{ijkl} d\varepsilon_{kl}$$

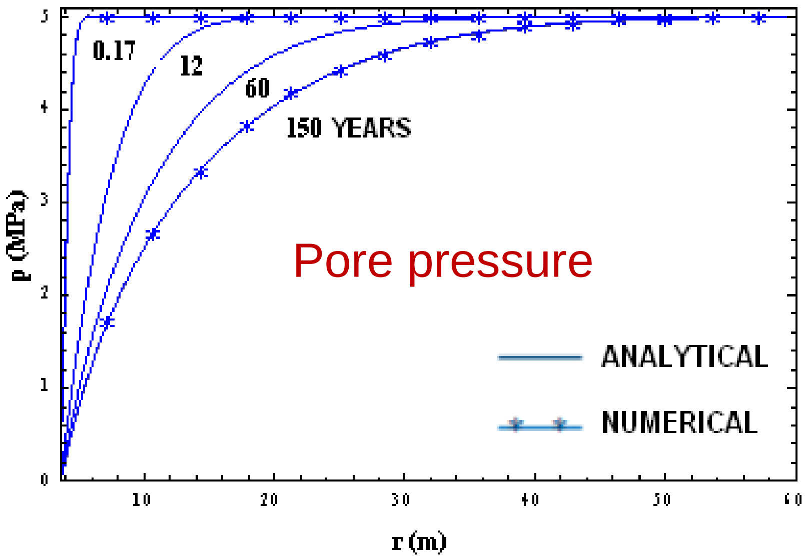
$$d\sigma_{ij} = D^{E}_{ijkl} d\varepsilon_{kl} - bp\delta_{ij}$$

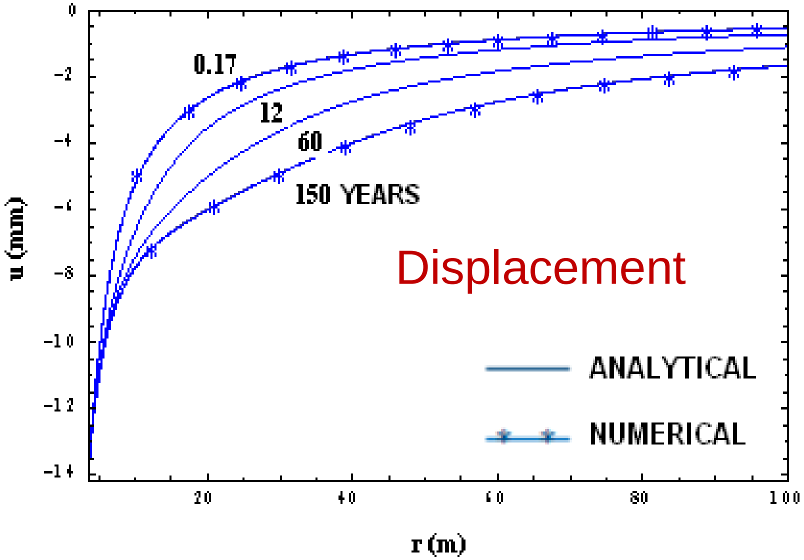
~0

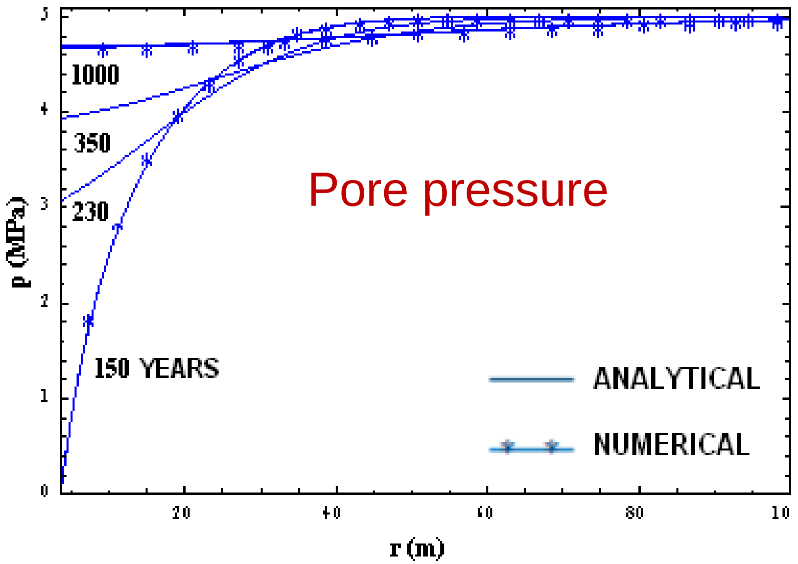
$$d\sigma_{ij} = D^{T}_{ijkl} d\varepsilon_{kl} - bp\delta_{ij}$$

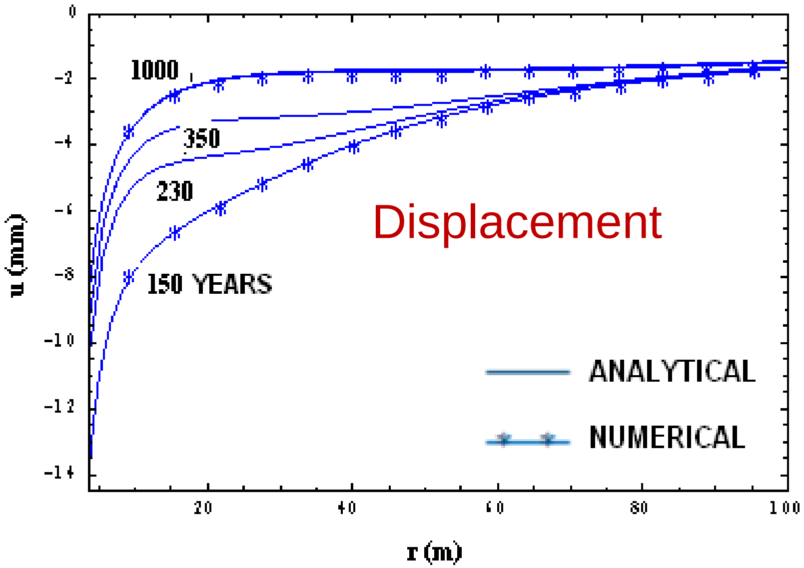
- Interface elements
  - liner - rock
  - backfill - liner
- possible gap opening

Comparisons : analytical & numerical Poroelasticity









Poro-viscoplastic damageable (iso) behavior law

- Strain partition

$$\dot{\boldsymbol{\varepsilon}} = \underbrace{\dot{\boldsymbol{\varepsilon}}^{e}}_{\text{elastic}} + \underbrace{\dot{\boldsymbol{\varepsilon}}^{vp}}_{\text{viscoplastic}}$$

- Damageable viscoplastic Lemaitre's law

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \frac{3}{2}\dot{\gamma}\frac{\boldsymbol{s}}{\sigma_{eq}} \qquad \text{with} \qquad \dot{\gamma} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{vp}:\dot{\boldsymbol{\varepsilon}}^{vp}}$$

$$\sigma_{eq} = \sqrt{\frac{3}{2}\boldsymbol{s}:\boldsymbol{s}} \qquad \dot{\gamma} = \frac{\dot{r}}{1-D} \qquad \dot{r} = \langle\frac{\sigma_{eq}}{(1-D)K\,r^{1/M}}\rangle^{N}$$

$$\dot{D} = \langle\frac{\sigma_{eq}}{A}\rangle^{R}(1-D)^{-k}$$

- \+ superposition → Damageable Poro-viscoplastic model

$$\sigma'_{ij} = \sigma_{ij} + bp\,\delta_{ij}$$

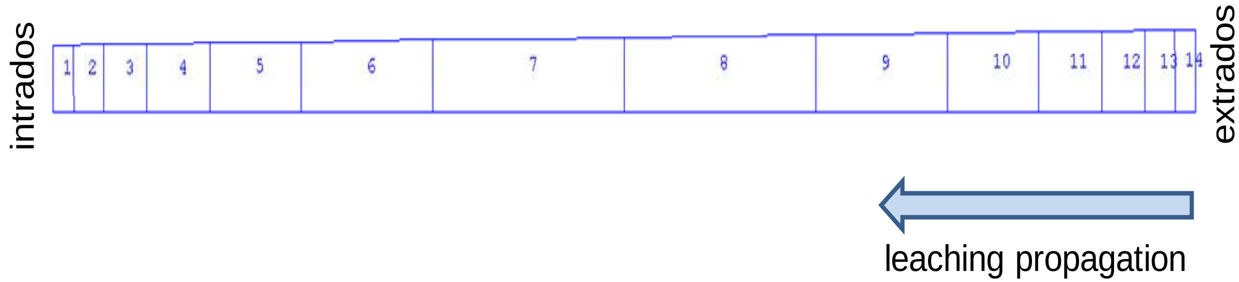
Parametric study relative to different kinetics of Lining support degradation

Normal & Accidental Scenarios

- Undamaged poroplastic liner
- Brittle poroplastic liner
- **Progressive leaching of the liner from outer to inner face**

Simplified model, $e_d(t) = e_d(\tau^0)\sqrt{\frac{t}{\tau^0}}$ focus on rock mass behaviour

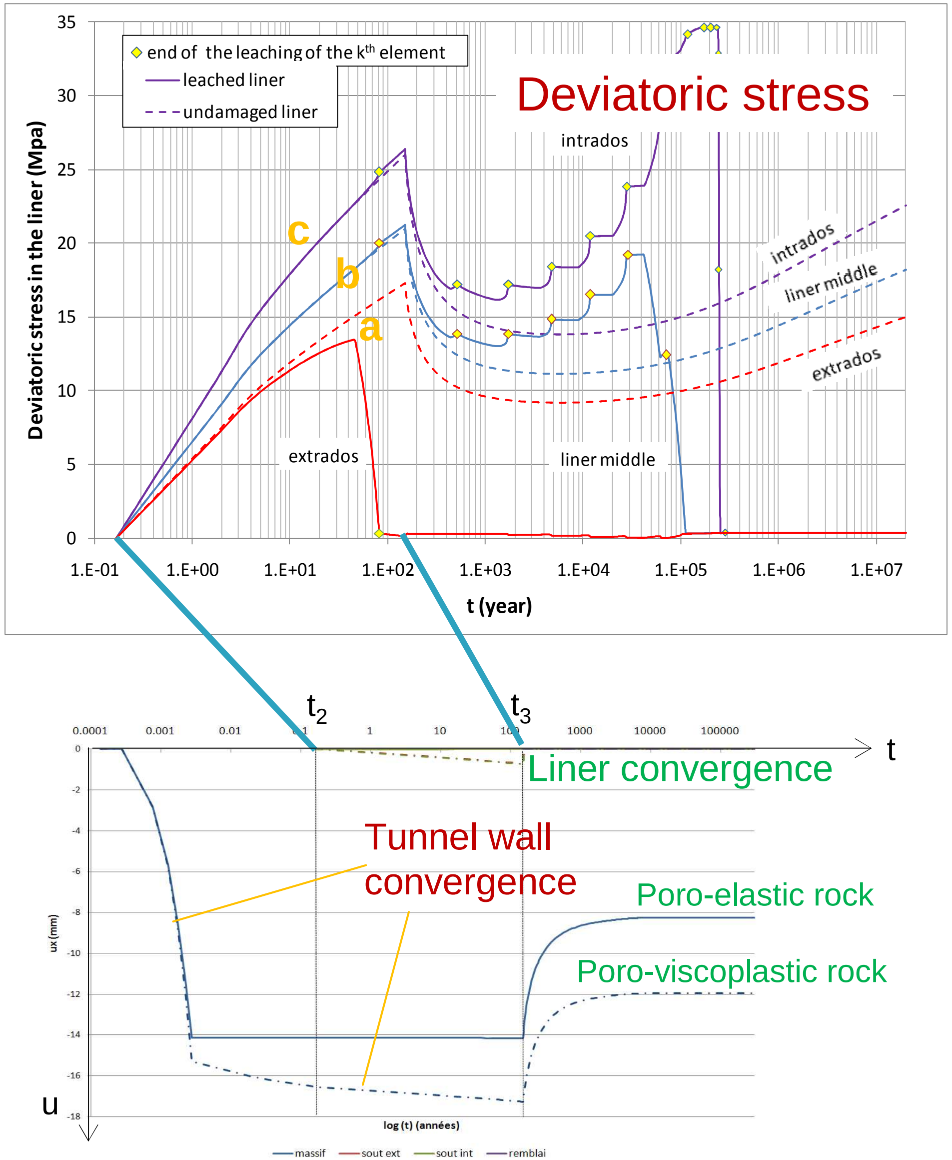
(Torrenti & al., 2008)

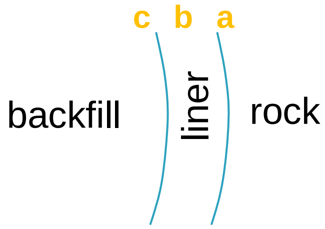


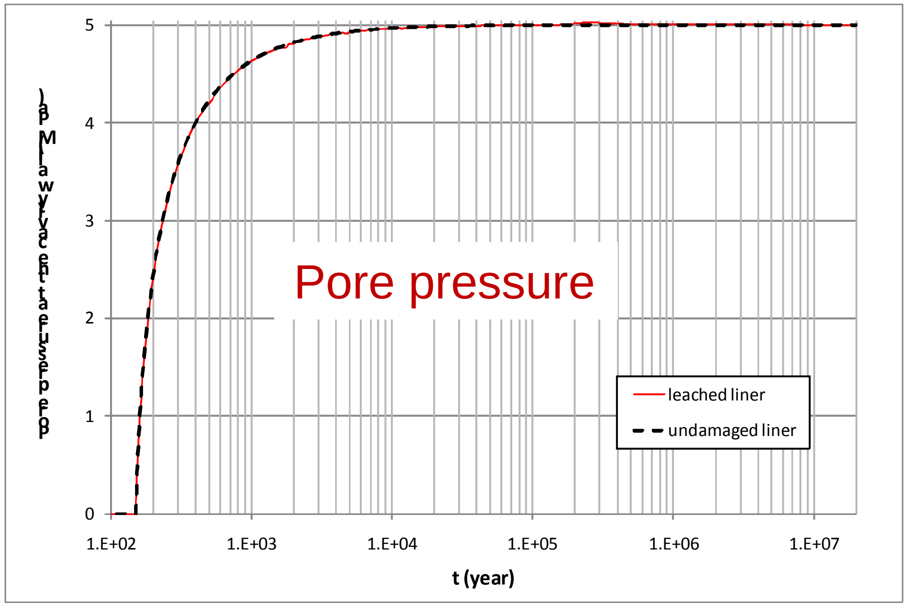
After leaching front: elastic moduli and strength → ~ 0

Lining support degradation

Progressive leaching front : extrados → intrados

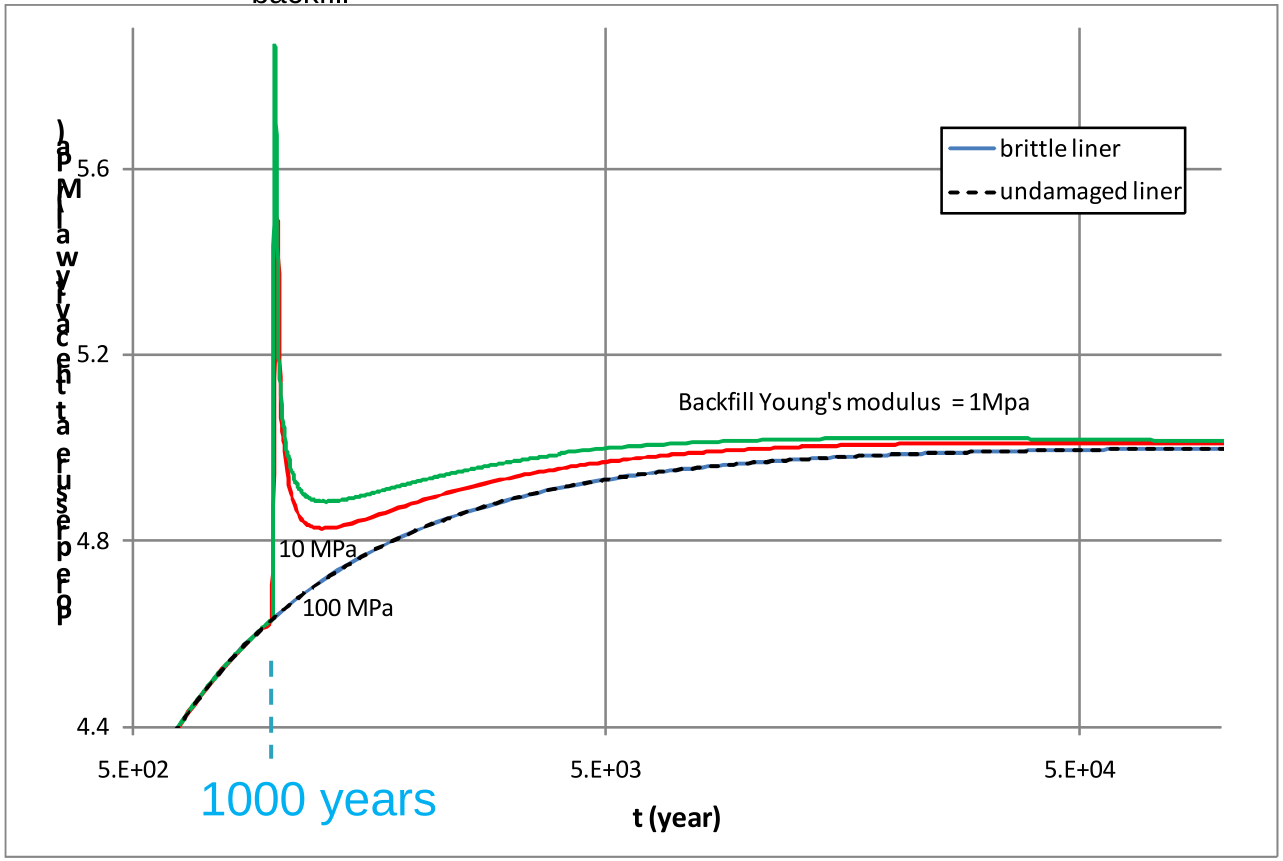






Backfill compaction

- Brittle poroplastic liner : sudden failure at t = 1000 years
  - $E_{backfill}$ = 100 MPa
  - $E_{backfill}$ = 10 MPa
  - $E_{backfill}$ = 1 MPa



Poro–viscoplastic behavior + anisotropic damage

$$\dot{\boldsymbol{\sigma}} = \mathbb{C}^{\sigma}(\boldsymbol{D}):\dot{\boldsymbol{\varepsilon}} - \dot{p_w}\boldsymbol{B}(\boldsymbol{D}) - \mathbb{C}(\boldsymbol{D}):\dot{\boldsymbol{\varepsilon}}^{vp}$$
$$\dot{\phi} = \boldsymbol{B}^{\phi}(\boldsymbol{D}):\dot{\boldsymbol{\varepsilon}} + \beta(\boldsymbol{D})\dot{p_w}$$

D = 2nd order tensor

$$f_d = f_d(\boldsymbol{\varepsilon}^+;\boldsymbol{D}) = \sqrt{\boldsymbol{\varepsilon}^+:\boldsymbol{\varepsilon}^+} - \left(r_0 + r_1 tr(\boldsymbol{D})\right)$$
$$\dot{\boldsymbol{D}} = \left(\frac{\boldsymbol{\varepsilon}^+ \otimes \boldsymbol{\varepsilon}^+}{r_1 tr(\boldsymbol{\varepsilon}^+)\|\boldsymbol{\varepsilon}^+\|}\right):\dot{\boldsymbol{\varepsilon}}^+$$

$$\boldsymbol{\varepsilon}^+ = \sum_{K=1}^{3} H(\varepsilon_K)\,\varepsilon_K\,\boldsymbol{e}_K \otimes \boldsymbol{e}_K = \mathbb{P}^+:\boldsymbol{\varepsilon}$$

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \sqrt{\frac{3}{2}}\dot{\gamma}_{vp}\,\frac{\frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}'}{\tilde{q}'} + \frac{\alpha(\boldsymbol{\delta}-\boldsymbol{D})}{3}}{\left\|\frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}'}{\tilde{q}'} + \frac{\alpha(\boldsymbol{\delta}-\boldsymbol{D})}{3}\right\|}$$

$$\tilde{\boldsymbol{\sigma}}' = \mathbb{M}(\boldsymbol{D}):\boldsymbol{\sigma}' = \mathbb{M}(\boldsymbol{D}):(\boldsymbol{\sigma} + p_w\boldsymbol{B}(\boldsymbol{D}))$$

$$\dot{\gamma}_{vp} = \sqrt{\frac{2}{3}}\left\langle\frac{\tilde{q}' + \alpha\,\tilde{\sigma}' - k}{K\gamma_{vp}^{\frac{1}{M}}}\right\rangle^{N}\left\|\frac{3}{2}\frac{\tilde{\tilde{\boldsymbol{s}}}'}{\tilde{q}'} + \frac{\alpha(\boldsymbol{\delta}-\boldsymbol{D})}{3}\right\|$$

→ 1D FEM code in C++ (Pereira, 2005)

Conclusions

- Multi-physical phenomena (HM-diffusion, creep, damage, leaching). Structural response depends on ratio between characteristic times

- Progressive (slow) leaching of liner

  Well compacted backfill (brutal liner failure)

  → no excess pore pressure

- Interesting result : complex transitional behavior at rock/liner & liner/backfill interfaces during hydraulic readjustment (possible theoretical gap-opening leading to hydraulic conductivity increase…)

- Perspective: improvements by accounting for:
  - anisotropic damage (on-going)
  - desaturation/resaturation phases